





FAKULTEIT INGENIERSWESE
FACULTY OF ENGINEERING



Stelsels en Seine 414 <i>Systems and Signals 414</i>		Eerste Toets 25 Maart 2014 <i>First Test</i> 25 March 2014	
Tydsduur <i>Duration</i>	2 h	Volpunte <i>Full marks</i>	50
Eksaminator: <i>Examiner:</i>		Mede-eksaminator(e): <i>Co-examiner(s):</i>	
T.R. Niesler		JA du Preez	

Sertifisering
Certification

Opgestel: Set:	Gemodereer: Moderated:
	
Eksaminator Examiner	Mede-eksaminator Co-examiner

Kandidaatinligting
Candidate's particulars

Van: <i>Surname:</i>	
Voorname: <i>First Names:</i>	
Studentenommer: <i>Student number:</i>	
Handtekening: <i>Signature:</i>	

Lees asseblief noukeurig die instruksies op die volgende bladsy.
Please read instructions on the next page carefully.

INSTRUKSIES

- *Vul u naam en studentenommer in soos aangedui op die voorblad van hierdie vraestel !*
- Lees die inligting op beide hierdie vraestel en die meegaande eksamenboek. Verskaf u gegewens op beide.
- Gee u antwoorde op die beskikbare plek onderaan elke vraag *op die vraestel*. **Die meegaande eksamenboek is beskikbaar net vir rofwerk en word nie gemerk nie.**
- U mag u voorgeskrewe handboek, Proakis & Manolakis sowel as die klasnotas soos in die lesings uitgedeel raadpleeg. Normale notas/kommentaar daarin is in orde. Geen verdere notas (ook nie in 'n sakrekenaar) word toegelaat nie.
- Toon en motiveer u redenasies altdy volledig. ***Punte sal afgetrek word indien dit nie gedoen word nie.*** Omskryf in woorde wat u probeer doen - dit tel in u guns indien u nie 'n berekening suksesvol deurvoer nie.
- Waar gegewens na u mening ontbreek, maak sinvolle, gemotiveerde aannames.
- Skryf met 'n pen. Sketse kan egter in potlood gemaak word.
- Plaas die voltooide vraestel in die rofwerkboek en handig beide (volledig) in.

INSTRUCTIONS

- *Fill in your name and student number in the space provided on the cover of this question paper!*
- Read the information on this question paper and on the accompanying examination book. Provide your details on both.
- Provide your answers in the space allocated after each question *on this question paper*. **The accompanying examination book is for rough-work only and will not be marked.**
- You may consult the prescribed handbook, Proakis & Manolakis as well as the handouts given in class. Normal notes/comments in it are acceptable. All further notes (also in a calculator) are forbidden.
- Always show and motivate your reasoning fully. ***Marks will be deducted for failing to do so.*** Describe what you are trying to do - this counts in your favour with unsuccessful calculations.
- If in your opinion any information is missing, make reasonable, motivated assumptions.
- Write with a pen. Sketches may be in pencil.
- Put the completed question paper inside the rough-work book and hand both (everything) in.

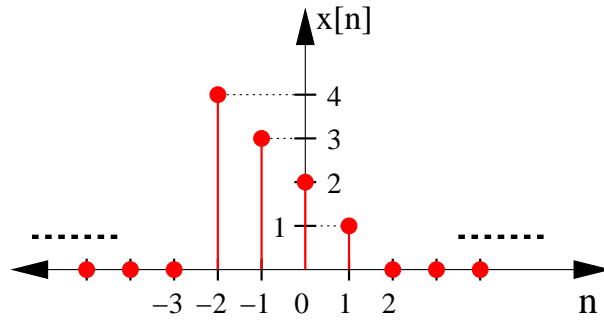
FOR MARKING PURPOSES ONLY

Question	1	2	3	4	5	6	7	8	9	10	11	12	Tot
Mark													
Check													

TOTAL:

Vraag 1 Beskou die volgende 4-monster diskrete-tyd sein $x[n]$:

Question 1 Consider the following 4-sample discrete-time signal $x[n]$:



Bepaal nou:

Now determine:

$$y[n] = x[3 - n] + x[n - 8]$$

en dui u antwoord in 'n skets aan.

(4)

and present your answer as a sketch.

(4)

Vraag 1 totaal: 4 punte.

Question 1 total: 4 marks.

Vraag 2 Bepaal die Fourier transform $X(\omega)$ van die volgende eindige-lengte diskrete-tyd sein $x[n]$:

Question 2 Determine the Fourier transform $X(\omega)$ of the following finite-length discrete-time signal $x[n]$:

$$x[n] = \begin{cases} \alpha^n & \text{vir/for } 0 \leq n \leq M \\ 0 & \text{elders/otherwise} \end{cases}$$

Vereenvoudig jou antwoord tot 'n breuk van polinome in $e^{j\omega}$. Wys en motiveer u bewerkings duidelik. *Wenk:* $\sum_{n=0}^{\infty} r^n = 1/(1-r)$ (5)

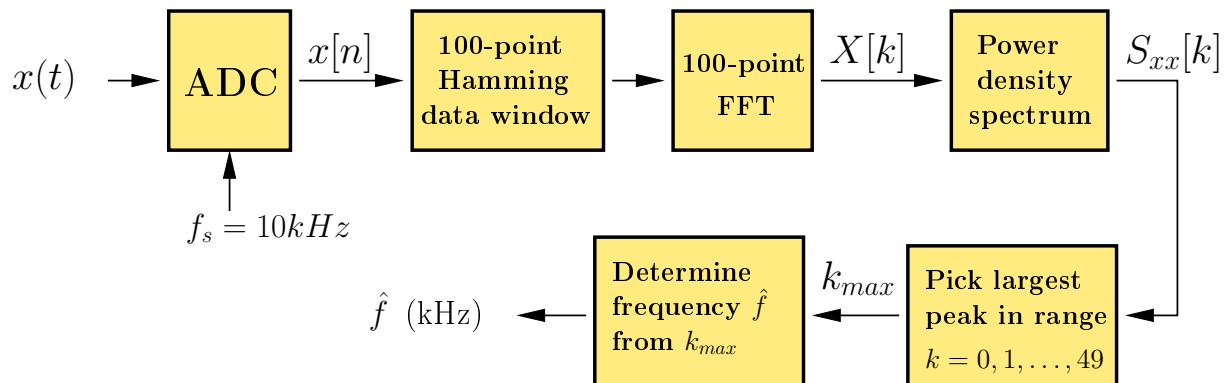
Simplify your result to a ratio of polynomials in $e^{j\omega}$. Show and motivate your working clearly. *Hint:* $\sum_{n=0}^{\infty} r^n = 1/(1-r)$ (5)

Vraag 2 totaal: 5 punte.

Question 2 total: 5 marks.

Vraag 3 Beskou die volgende blok diagram wat 'n stelsel beskrywe waarmee die frekwensie \hat{f} van 'n kontinue-tyd sein $x(t)$ bepaal kan word, waar t die tyd in sekondes is en \hat{f} die afgeskatte frekwensie in kHz.

Question 3 Consider the following block diagram which describes a system that can be used to determine the frequency \hat{f} of a continuous-time signal $x(t)$, where t is the time in seconds and \hat{f} the estimated frequency in kHz.



Aanvaar nou dat die intree sein gegee word deur: Now assume that the input signal is given by:

$$x(t) = \cos(14000\pi t)$$

en dat:

and that:

$$x[n] = x(n/f_s)$$

a) Bepaal nou die afgeskatte frekwensie \hat{f} . Wys en motiveer u bewerkings duidelik. (4)

a) Now determine the estimated frequency \hat{f} . Show and motivate your working clearly. (4)

Vraag 3 (vervolg)

berekenings.

Addisionele ruimte vir

Question 3 (continued)

working.

Additional space for

b) Hoe sou u die stelsel verbeter?

(2)

b) How would you improve the system?

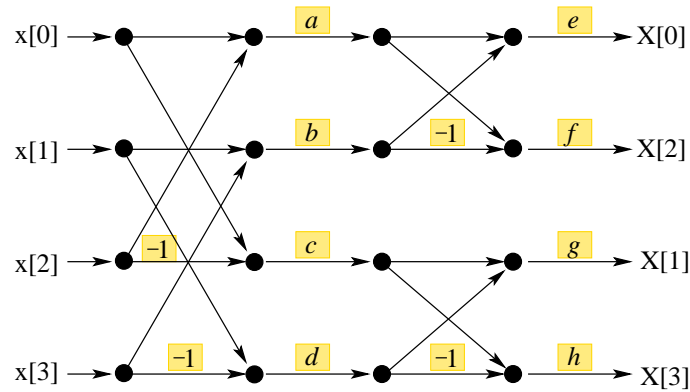
(2)

Vraag 3 totaal: 6 punte.

Question 3 total: 6 marks.

Vraag 4 Die volgende figuur toon die butterfly-berekeninge van 'n 4-punt desimasie-in-frekwensie FFT, met $x[n]$ as intree tyd-domein sein en $X[k]$ die uitree frekwensie-domein sein.

Question 4 The following figure shows the butterfly computations of a 4-point decimation-in-frequency FFT, where $x[n]$ is the input time-domain signal and $X[k]$ the output frequency domain signal.



Die gewigfaktore van alle takke is eenheid tensy anders aangedui. Bepaal die waardes van die gewigfaktore a , b , c , d , e , f , g en h . Wys en motiveer u bewerkings duidelik. (5)

The weighting on each branch is unity except when a value is indicated. Determine the values of the weight factors a , b , c , d , e , f , g and h . Show and motivate your working clearly. (5)

Vraag 5 Gebruik die definisie van diskrete konvolusie om te bewys dat diskrete konvolusie distributief is oor sommasie, d.w.s. bewys dat:

$$x[n] * [h_1[n] + h_2[n]] = x[n] * h_1[n] + x[n] * h_2[n]$$

waar $x[n]$, $h_1[n]$ en $h_2[n]$ almal energieseine is. Wys en motiveer u bewerkings duidelik. (4)

Question 5 From the definition of discrete convolution, show that discrete convolution is distributive over addition, i.e. that:

where $x[n]$, $h_1[n]$ and $h_2[n]$ are all energy signals. Show and motivate your working clearly. (4)

Vraag 6 Beskou die volgende diskrete-tyd sein $x[n]$:

Question 6 Consider the following discrete-time signal $x[n]$:

$$x[n] = 2 \cos\left(\frac{2\pi n}{5}\right) - \cos\left(\frac{8\pi n}{5}\right) + \sin\left(\frac{12\pi n}{5}\right)$$

a) Bepaal en skets die 10-punt drywingsdigtheid-spektrum $S_{xx}[k]$ van die eerste 10 monsters van $x[n]$, d.w.s.: $x[0], x[1], \dots, x[9]$. Wys en motiveer u bewerkings duidelik. (10)

a) Sketch the 10-point power density spectrum $S_{xx}[k]$ using the first 10 samples of $x[n]$, i.e.: $x[0], x[1], \dots, x[9]$. Show and motivate your working clearly. (10)

b) Gebruik u antwoord op (a) om die drywing in die sein $x[n]$ te bereken. Toon en motiveer u berekening duidelik. (2)

b) Use your answer to (a) to calculate the power of the signal $x[n]$. Show and motivate your calculations. (2)

Vraag 6 totaal: 12 punte.

Question 6 total: 12 marks.

Vraag 7 Beskou twee arbitrêre maar reële diskrete-tyd seine $x[n]$ en $y[n]$ waar $x[n]$ uit N_x monsters bestaan en $y[n]$ uit N_y monsters bestaan. Aanvaar verder dat $N_x > N_y$.

a) Bepaal 'n geslote-vorm uitdrukking, in terme N_x en N_y , vir die aantal reële vermenigvuldigings wat benodig word om $x[n] * y[n]$ te bereken wanneer die definisie van diskrete konvolusie direk geïmplementeer word. Wys en motiveer u bewerkings duidelik. *Wenk:* $\sum_{n=0}^{N-1} n = N(N-1)/2$
(5)

Question 7 Consider two arbitrary but real discrete-time signals $x[n]$ and $y[n]$ where $x[n]$ consists of N_x samples and $y[n]$ of N_y samples. Assume furthermore that $N_x > N_y$.

a) Determine a closed-form expression, in terms of N_x and N_y , for the number of real multiplications required to calculate $x[n] * y[n]$ when implementing the definition of discrete convolution directly. Show and motivate your working clearly. *Hint:* $\sum_{n=0}^{N-1} n = N(N-1)/2$ (5)

Vraag 7 (vervolg)

b) Bepaal 'n geslote-vorm uitdrukking, in terme N_x en N_y , vir die aantal reële vermenigvuldigings wat benodig word om die diskrete konvolusie $x[n] * y[n]$ te bereken wanneer van die radiks-2 FFT gebruik gemaak word. (5)

c) Aanvaar nou dat $N_x = 256$. Vir watter waardes van N_y is (b) meer doeltreffend? (4)

Question 7 (continued)

b) Determine a closed-form expression, in terms of N_x and N_y , for the number of real multiplications required to calculate the discrete convolution $x[n] * y[n]$ when using the radix-2 FFT. (5)

c) Now assume $N_x = 256$. For which values of N_y is (b) more efficient? (4)

Vraag 7 totaal: 14 punte.**Question 7 total: 14 marks.****Vraestel totaal: 50****Question-paper total: 50**