





**FAKULTEIT INGENIERSWESE**  
**FACULTY OF ENGINEERING**



Stelsels en Seine 414 <i>Systems and Signals 414</i>		Eerste Toets 30 Maart 2012 <i>First Test</i> 30 March 2012	
Tydsduur <i>Duration</i>	<b>1.5 h</b>	Volpunte <i>Full marks</i>	<b>50</b>
Eksaminator: <i>Examiner:</i>		Mede-eksaminator(e): <i>Co-examiner(s):</i>	
T.R. Niesler		J.A. du Preez	

**Sertifisering**  
***Certification***

Opgestel: Set:	Gemodereer: Moderated:
	
Eksaminator Examiner	Mede-eksaminator Co-examiner

**Kandidaatinligting**  
***Candidate's particulars***

Van: <i>Surname:</i>	
Voorname: <i>First Names:</i>	
Studentenommer: <i>Student number:</i>	
Handtekening: <i>Signature:</i>	

Lees asseblief noukeurig die instruksies op die volgende bladsy.  
*Please read instructions on the next page carefully.*

## INSTRUKSIES

- *Vul u naam en studentenommer in soos aangedui op die voorblad van hierdie vraestel !*
- Lees die inligting op beide hierdie vraestel en die meegaande eksamenboek. Verskaf u gegewens op beide.
- Gee u antwoorde op die beskikbare plek onderaan elke vraag *op die vraestel*. **Die meegaande eksamenboek is beskikbaar net vir rofwerk en word nie gemerk nie.**
- U mag u voorgeskrewe handboek, Proakis & Manolakis sowel as die klasnotas soos in die lesings uitgedeel raadpleeg. Normale notas/kommentaar daarin is in orde. Geen verdere notas (ook nie in 'n sakrekenaar) word toegelaat nie.
- Toon en motiveer u redenasies altdy volledig. ***Punte sal afgetrek word indien dit nie gedoen word nie.*** Omskryf in woorde wat u probeer doen - dit tel in u guns indien u nie 'n berekening suksesvol deurvoer nie.
- Waar gegewens na u mening ontbreek, maak sinvolle, gemotiveerde aannames.
- Skryf met 'n pen. Sketse kan egter in potlood gemaak word.
- Plaas die voltooide vraestel in die rofwerkboek en handig beide (volledig) in.

## INSTRUCTIONS

- *Fill in your name and student number in the space provided on the cover of this question paper!*
- Read the information on this question paper and on the accompanying examination book. Provide your details on both.
- Provide your answers in the space allocated after each question *on this question paper*. **The accompanying examination book is for rough-work only and will not be marked.**
- You may consult the prescribed handbook, Proakis & Manolakis as well as the handouts given in class. Normal notes/comments in it are acceptable. All further notes (also in a calculator) are forbidden.
- Always show and motivate your reasoning fully. ***Marks will be deducted for failing to do so.*** Describe what you are trying to do - this counts in your favour with unsuccessful calculations.
- If in your opinion any information is missing, make reasonable, motivated assumptions.
- Write with a pen. Sketches may be in pencil.
- Put the completed question paper inside the rough-work book and hand both (everything) in.

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### FOR MARKING PURPOSES ONLY

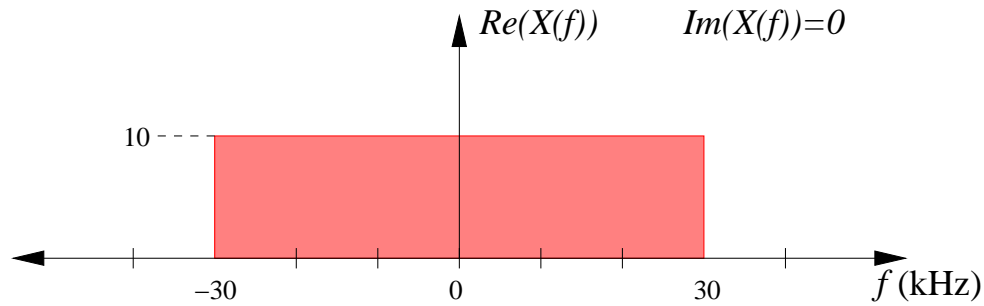
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Question	1	2	3	4	5	6	7	8	9	10	11	12	Tot
Mark													
Check													

TOTAL:

**Vraag 1** Beskou die kontinue-tyd sein  $x(t)$  met die volgende spektrum (Fourier transform)  $X(f)$ .

**Question 1** Consider the continuous-time signal  $x(t)$  that has the following spectrum (Fourier transform)  $X(f)$ .



a) 'n Diskrete-tyd sein  $x[n]$  word verkry deur die kontinue-tyd sein  $x(t)$  teen 'n monsterfrekwensie van  $f_s = 70\text{kHz}$  te monster. Skets die spektrum  $X(f_\omega)$  van die gemonsterde sein  $x[n]$  oor die interval  $-1 < f_\omega < 1$ , waar  $f_\omega$  die frekwensie in siklusse/monster is. Dui amplitudes en frekwensies deeglik aan. (4)

a) A discrete-time signal  $x[n]$  is obtained by sampling the continuous-time signal  $x(t)$  at a sampling frequency  $f_s = 70\text{kHz}$ . Sketch the spectrum  $X(f_\omega)$  of the sampled signal  $x[n]$  over the interval  $-1 < f_\omega < 1$ , where  $f_\omega$  is the frequency in cycles/sample. Label amplitudes and frequencies thoroughly. (4)

**Vraag 1 (vervolg)**

b) Die sein  $x(t)$  word nou teen 'n monster-frekwensie van  $f_s = 40\text{kHz}$  gemonster. Skets die spektrum  $X(f_\omega)$  van die gemonsterde sein  $x[n]$  oor die interval  $-1 < f_\omega < 1$  in hierdie geval. Dui amplitudes en frekwensies deeglik aan. (6)

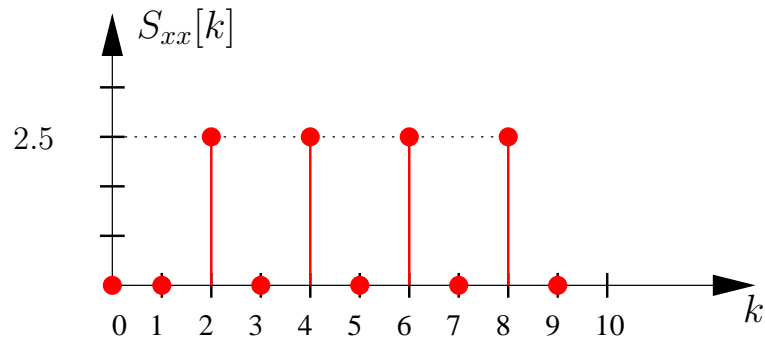
**Question 1 (continued)**

b) The signal  $x(t)$  is now sampled at  $f_s = 40\text{kHz}$ . Sketch the spectrum  $X(f_\omega)$  of the sampled signal  $x[n]$  over the interval  $-1 < f_\omega < 1$  for this case. Label amplitudes and frequencies thoroughly. (6)

**Vraag 1 totaal: 10 punte.****Question 1 total: 10 marks.**

**Vraag 2** Beskou die skets van die 10-punt drywingsdigtheidspektrum  $S_{xx}[k]$  van 'n periodesiese diskrete-tyd sein  $x[n]$ .

**Question 2** Consider the graph of the 10-point power density spectrum  $S_{xx}[k]$  of a periodic discrete-time signal  $x[n]$ .



Watter van die volgende kontinue-tyd seine  $x(t)$  sou NIE lei na die drywingsdigtheidspektrum hierbo nie as dit gemonster word teen  $f_s = 25\text{kHz}$ ? Daar mag meer as een wees! Toon en motiveer u berekeninge. (12)

Which of the following continuous-time signals  $x(t)$  would NOT lead to the above power density spectrum when sampled at  $f_s = 25\text{kHz}$ ? There may be more than one! Show and motivate your calculations. (12)

- (a)  $x(t) = \sin(20000\pi t) + \cos(30000\pi t)$
- (b)  $x(t) = \cos(30000\pi t) + \sin(40000\pi t)$
- (c)  $x(t) = \cos(20000\pi t) + \sin(60000\pi t)$
- (d)  $x(t) = \sin(20000\pi t) + \cos(70000\pi t)$

**Vraag 2 (vervolg)**

*Addisionele ruimte vir berekenings.*

**Question 2 (continued)**

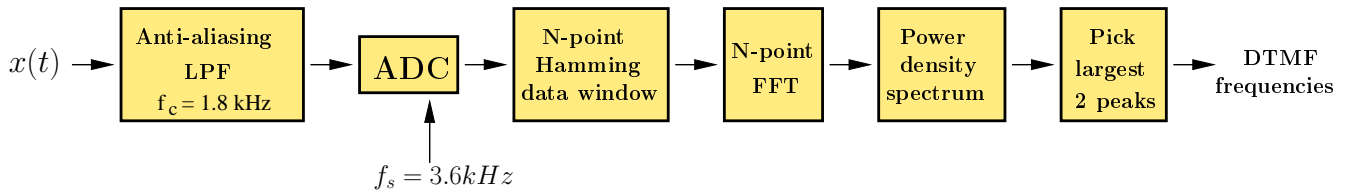
*Additional space for working.*

**Vraag 2 totaal: 12 punte.**

**Question 2 total: 12 marks.**

**Vraag 3** Die volgende figuur illustreer 'n stelsel wat gebruik kan word om *dual-tone multi-frequency (DTMF, multifrekwensie tone)* seine to ontleed.

**Question 3** The following figure illustrates a system that can be used to decode dual-tone multi-frequency (DTMF) signals.



Bepaal die minimum lengte van die Hamming venster (in monsters) wat benodig word om volle hooflob-skeiding te verseker in die drywingsdigtheidspektrum tussen alle DTMF frekwensie komponente. Toon en motiveer u berekening. (6)

Determine the minimum length of the Hamming window (in samples) that is necessary to ensure full main-lobe separation in the power density spectrum between all DTMF frequency components. Show and motivate your calculations. (6)

**Vraag 4** Beskou die volgende sein wat 'n diskrete-tyd sein  $x[n]$  eers opmonster met 'n faktor 2 en dan die resulterende sein afmonster met 'n faktor 3.

**Question 4** Consider the following system, which first upsamples a discrete-time signal  $x[n]$  by a factor 2 and then downsamples the resulting signal by a factor 3.



Aanvaar nou dat die intreesein  $x[n]$  gegee word deur:

Assume now that the input signal  $x[n]$  is given by:

$$x[n] = \cos\left(\frac{4\pi n}{5}\right)$$

Bepaal en skets die 10-punt DFT van die diskrete-tyd sein  $y[n]$  soos aangedui in die figuur. Toon en motiveer u berekeninge. (8)

Determine and sketch the 10-point DFT of the discrete-time signal  $y[n]$  as indicated in the figure. Show and motivate your calculations. (8)



**Vraag 4 (vervolg)**  
berekenings.

*Additionele ruimte vir*

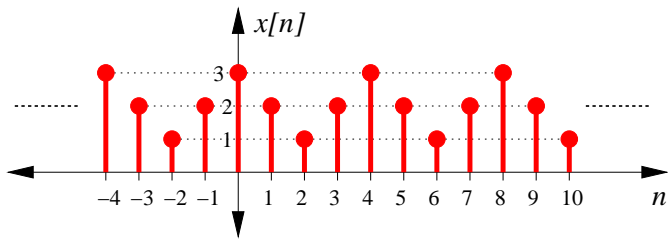
**Question 4 (continued)**  
working.

*Additional space for*

**Vraag 4 totaal: 8 punte.**

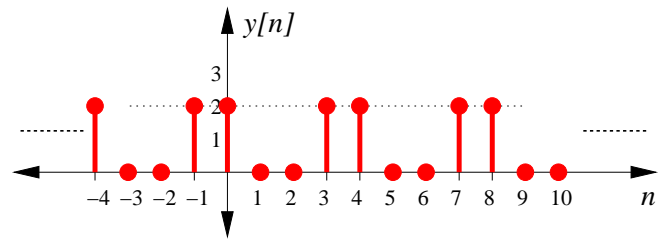
**Question 4 total: 8 marks.**

**Vraag 5** Beskou die volgende twee periodesiese diskrete-tyd seine  $x[n]$  en  $y[n]$ .



Bepaal en skets die sirkulêre konvolusie  $g[n]$  van hierdie twee seine oor die bereik  $0 \leq n \leq 7$ . Toon en motiveer u berekeninge. (6)

**Question 5** Consider the following two periodic discrete-time signals  $x[n]$  and  $y[n]$ .



Determine and sketch the circular convolution  $g[n]$  of these two signals for the range  $0 \leq n \leq 7$ . Show and motivate your calculations. (6)

**Vraag 6** Beskou die vergelyking van die volgende diskrete-tyd sein  $x[n]$ :

**Question 6** Consider the equation of the following discrete-time signal  $x[n]$ :

$$x[n] = \sin\left(\frac{\pi n}{4}\right)$$

Bepaal 'n geslote-vorm uitdrukking vir die outokorrelasie  $r_{xx}[i]$  van  $x[n]$  en plot u antwoord oor die bereik  $0 \leq i \leq 7$ . Toon en motiveer u berekening. *Wenk: maak gebruik van frekwensie-domein metodes.* (8)

Determine a closed-form expression for the autocorrelation  $r_{xx}[i]$  of  $x[n]$  and plot your result over the range  $0 \leq i \leq 7$ . Show and motivate your calculations. *Hint: make use of frequency domain methods.* (8)

Vraag 6 totaal: 8 punte.

Question 6 total: 8 marks.

Vraestel totaal: 50

Question-paper total: 50