FAKULTEIT INGENIERSWESE
FACULTY OF ENGINEERING

Stelsels en Seine 414 *Systems and Signals 414*	Tweede Toets 5 Junie 2014 *Second Test* *5 June 2014*

Tydsduur *Duration*	3 h	Volpunte *Full marks*	90

Eksaminator: *Examiner:*	Mede-eksaminator(e): *Co-examiner(s):*
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Sertifisering
Certification

Opgestel: Set:	Gemodereer: Moderated:
Eksaminator Examiner	Mede-eksaminator Co-examiner

Kandidaatinligting
Candidate's particulars

Van: *Surname:*	
Voorname: *First Names:*	
Studentenommer: *Student number:*	
Handtekening: *Signature:*	

Lees asseblief noukeurig die instruksies op die volgende bladsy.
Please read instructions on the next page carefully.

INSTRUKSIES	INSTRUCTIONS
• ***Vul u naam en studentenommer in soos aangedui op die voorblad van hierdie vraestel !***	• ***Fill in your name and student number in the space provided on the cover of this question paper!***
• Lees die inligting op beide hierdie vraestel en die meegaande eksamenboek. Verskaf u gegewens op beide.	• Read the information on this question paper and on the accompanying examination book. Provide your details on both.
• Gee u antwoorde op die beskikbare plek onderaan elke vraag *op die vraestel.* **Die meegaande eksamenboek is beskikbaar net vir rofwerk en word nie gemerk nie.**	• Provide your answers in the space allocated after each question *on this question paper.* **The accompanying examination book is for rough-work only and will not be marked.**
• U mag u voorgeskrewe handboek, Proakis & Manolakis sowel as die klasnotas soos in die lesings uitgedeel raadpleeg. Normale notas/kommentaar daarin is in orde. Geen verdere notas (ook nie in 'n sakrekenaar) word toegelaat nie.	• You may consult the prescribed handbook, Proakis & Manolakis as well as the handouts given in class. Normal notes/comments in it are acceptable. All further notes (also in a calculator) are forbidden.
• Toon en motiveer u redenasies altyd volledig. ***Punte sal afgetrek word indien dit nie gedoen word nie***. Omskryf in woorde wat u probeer doen - dit tel in u guns indien u nie 'n berekening suksesvol deurvoer nie.	• Always show and motivate your reasoning fully. ***Marks will be deducted for failing to do so***. Describe what you are trying to do - this counts in your favour with unsuccessful calculations.
• Waar gegewens na u mening ontbreek, maak sinvolle, gemotiveerde aannames.	• If in your opinion any information is missing, make reasonable, motivated assumptions.
• Skryf met 'n pen. Sketse kan egter in potlood gemaak word.	• Write with a pen. Sketches may be in pencil.
• Plaas die voltooide vraestel in die rofwerkboek en handig beide (volledig) in.	• Put the completed question paper inside the rough-work book and hand both (everything) in.

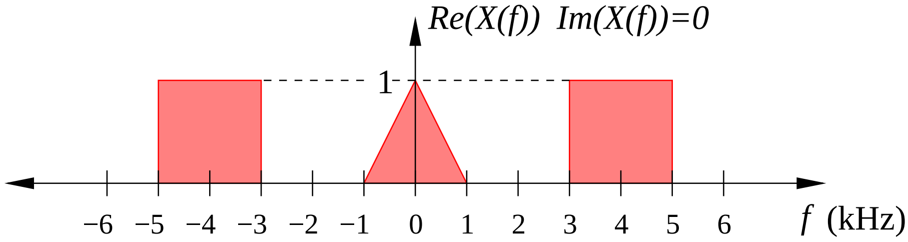
FOR MARKING PURPOSES ONLY

Question	1	2	3	4	5	6	7	8	9	10	11	12	**Tot**
Mark													
Check													

TOTAL:

Vraag 1 Beskou die kontinue-tyd sein $x(t)$ met die volgende spektrum (Fourier transform) $X(f)$.

Question 1 Consider the continuous-time signal $x(t)$ that has the following spectrum (Fourier transform) $X(f)$.



a) 'n Diskrete-tyd sein $x[n]$ word verkry deur die kontinue-tyd sein $x(t)$ teen 'n monsterfrekwensie van $f_s = 10\text{kHz}$ te monster. Skets die spektrum $X(f_\omega)$ van die gemonsterde sein $x[n]$ oor die interval $-1 < f_\omega < 1$, waar f_ω die frekwensie in siklusse/monster is. Dui asse, amplitudes en frekwensies deeglik aan. Toon en motiveer u berekeninge duidelik. (4)

a) A discrete-time signal $x[n]$ is obtained by sampling the continuous-time signal $x(t)$ at a sampling frequency $f_s = 10\text{kHz}$. Sketch the spectrum $X(f_\omega)$ of the sampled signal $x[n]$ over the interval $-1 < f_\omega < 1$, where f_ω is the frequency in cycles/sample. Label axes, amplitudes and frequencies thorougly. Show and motivate your calculations. (4)

Vervolg ...

Continued ...

Vraag 1 (vervolg)

b) 'n Nuwe diskrete-tyd sein $x[n]$ word verkry deur die kontinue-tyd sein $x(t)$ teen 'n laer monsterfrekwensie van $f_s = 6$kHz te monster. Skets die spektrum $X(f_\omega)$ van die gemonsterde sein $x[n]$ oor die interval $-1 < f_\omega < 1$, waar f_ω die frekwensie in siklusse/monster is. Dui asse, amplitudes en frekwensies deeglik aan. Toon en motiveer u berekeninge duidelik. (6)

Question 1 (continued)

b) A new discrete-time signal $x[n]$ is obtained by sampling the continuous-time signal $x(t)$ at the lower sampling frequency $f_s = 6$kHz. Sketch the spectrum $X(f_\omega)$ of the sampled signal $x[n]$ over the interval $-1 < f_\omega < 1$, where f_ω is the frequency in cycles/sample. Label axes, amplitudes and frequencies thorougly. Show and motivate your calculations. (6)

Vervolg ...

Continued ...

Vraag 1 (vervolg)

c) Bepaal 'n stategie waarmee die oorspronklike sein $x(t)$ herwin kan word uit die sein in (b). Dui u voorgestelde strategie in blokdiagram vorm aan, en motiveer alle stappe duidelik. (8)

Vraag 1 totaal: 18 punte.

Question 1 (continued)

c) Devise a strategy by means of which the original signal $x(t)$ can be reconstructed from the signal in (b). Show your proposed strategy as a block diagram, and motivate all steps clearly. (8)

Question 1 total: 18 marks.

Vraag 2 'n Diskrete-tyd sein $x[n]$ word verkry deur 'n kontinue-tyd sein $x(t)$ teen 'n monsterfrekwensie van 40kHz te monster. Die sein $x(t)$ bestaan uit 'n enkele sinusvormige komponent, waarvan die frekwensie lineêr styg van 0Hz tot 20kHz binne twee sekondes, en daarna lineêr weer daal tot by 0Hz binne dieselfde tyd interval. Die sein $x[n]$ word geanaliseer met behulp van 'n spektrogram met die doel om die oombliklike frekwensie elke 2ms te bepaal tot binne 200Hz.

a) Analiese geskied eers deur middel van 'n spektrogram waar die raamlengte $N = 32$ monsters is, die raam-inkrement (frame skip) $R = 16$ monsters is, en 'n Hamming venster op elke raam toegepas word. Is hierdie geskikte waardes vir die analiese? Verduidelik hoekom (nie).Wys en motiveer u bewerkings duidelik. (4)

Vervolg ...

Question 2 A discrete-time signal $x[n]$ is obtained by sampling a continuous-time signal $x(t)$ at a sampling frequency of 40kHz. The signal $x(t)$ consists of a single sinusoid, increasing linearly in frequency from 0Hz to 20kHz in the space of 2 seconds, then decreasing linearly back to 0Hz within the same space of time. The signal $x[n]$ is analysed using a spectrogram with the aim of determining the instantaneous frequency every 2ms to within 200Hz.

a) Analysis first occurs using a spectrogram in which the frame length is $N = 32$ samples, the frame skip is $R = 16$ samples, and a Hamming window is applied to every frame. Are these appropriate values for the analysis? Explain why (not). Show and motivate your working clearly. (4)

Continued ...

Vraag 2 (vervolg)

b) Analiese geskied nou deur middel van 'n spektrogram waar die raamlengte $N = 2048$ monsters is, die raam-inkrement (frame skip) $R = 256$ monsters is, en 'n Hamming venster op elke raam toegepas word. Is hierdie geskikte waardes vir die analiese? Verduidelik hoekom (nie). Wys en motiveer u bewerkings duidelik. (4)

c) Indien 'n Hamming venster op elke raam toegepas word, bepaal ideale waardes vir N and R. Wys en motiveer u bewerkings duidelik. (4)

Vraag 2 totaal: 12 punte.

Question 2 (continued)

b) Analysis now occurs using a spectrogram in which the frame length is $N = 2048$ samples, the frame skip is $R = 256$ samples, and a hamming window is applied to every fame. Are these appropriate values for the analysis? Show and motivate your working clearly. (4)

c) Assuming that a Hamming window is applied to every frame, determine ideal values for N and R. Show and motivate your working clearly. (4)

Question 2 total: 12 marks.

Vraag 3 Aanvaar $H(z)$ is 'n lineêre tydinvariante eindige-impulsweergawe (FIR) filter met 'n 950 koëffisiënte $b_0 \ldots b_{949}$. Hierdie filter moet op 'n DSP verwerker geïmplementeer word met behulp van die *overlap and add* (OLA) metode en radiks-2 FFTs. Bepaal die optimale bloklengte L wat gebruik moet word om die aantal wisselpunt vermenigvuldigings per uittree monster te minimeer. (6)

Question 3 Assume $H(z)$ is a linear time-invariant finite impulse response (FIR) filter with a 950 coefficients $b_0 \ldots b_{949}$. This filter must be implemented on a DSP processor using the overlap and add (OLA) method and radix-2 FFTs. Determine the optimal block length L that must be used to minimise the number of required floating-point multiplications per output sample. (6)

Vraag 3 totaal: 6 punte.

Question 3 total: 6 marks.

Vraag 4 Beskou die volgende z-transform $X(z)$:

Question 4 Consider the following z-transform $X(z)$:

$$H(z) = \frac{z^{-1} + z^{-2} + z^{-3} - z^{-5} - z^{-7} - z^{-9}}{1 - z^{-1}}$$

Plot die kousale sein $X[n]$ met hierdie z-transform. (5)

Plot the causal signal $x[n]$ with this z-transform. (5)

Vraag 4 totaal: 5 punte.

Question 4 total: 5 marks.

Vraag 5 Beskou die diskrete-tyd stelsel wat deur die volgende lineêre konstante-koëffisiënt verskilvergelyking beskryf word:

Question 5 Consider the discrete-time system described by the following linear constant-coefficient difference equation :

$$y[n] - 1.5y[n-1] + 0.5y[n-2] = 0$$

Bepaal 'n geslote-vorm uitdrukking vir die weergawe $y[n]$ van die stelsel, met $n \geq 0$, gegee die volgende begintoestande.

Determine a closed-form expression for the response $y[n]$ of the system, with $n \geq 0$, given the following initial condittions.

$$\begin{aligned} y[-1] &= 1 \\ y[-2] &= 0 \end{aligned}$$

Skets u resultaat, en dui spesifiek die waardes van $y[0]$, $y[1]$ en $\lim_{n\to\infty} y[n]$ aan. Wys en motiveer u bewerkings duidelik. (10)

Sketch your result, indicating specifically the values of $y[0]$, $y[1]$ and $\lim_{n\to\infty} y[n]$. Show and motivate your working clearly. (10)

Vervolg ...

Continued ...

Vraag 5 (vervolg) *Addisionele ruimte vir berekenings.*

Question 5 (continued) *Additional space for working.*

Vraag 5 totaal: 10 punte.

Question 5 total: 10 marks.

Vraag 6 Beskou 'n diskrete-tyd lineêre tyd-invariante (LTI) stelsel met die volgende impulsweergawe $h[n]$:

Question 6 Consider a discrete-time linear time-invariant (LTI) system with the following impulse reponse $h[n]$:

$$h[n] = \{\underset{\uparrow}{1}, 0, -1\}$$

Aanvaar nou dat die volgende intree $x[n]$ op die stelsel aangelê word:

Assume now that this system is presented with the following input $x[n]$:

$$x[n] = \{\underset{\uparrow}{1}, 2, 3, 2, 1\}$$

a) Bereken die respons $y[n]$ van die stelsel tot hierdie intree. Wys en motiveer u bewerkings duidelik. (2)

a) Calculate the response $y[n]$ of the system to this input. Show and motivate your working clearly. (2)

b) Bepaal 'n reële en geslote-vorm uitdrukking vir die amplitude van die frekwensieweergawe $|H(e^{j\omega}|$. Skets hierdie amplitudeweergawe oor die interval $0 \leq \omega \leq 2\pi$. Wys en motiveer u bewerkings duidelik. (4)

b) Determine a real and closed-form expression for the magnitude of the frequency response $|H(e^{j\omega}|$. Sketch this magnitude over the interval $0 \leq \omega \leq 2\pi$. Show and motivate your working clearly. (4)

Vervolg ...

Continued ...

Vraag 6 (vervolg) *Addisionele ruimte vir berekenings.*

Question 6 (continued) *Additional space for working.*

c) Bepaal 'n reële en geslote-vorm uitdrukking vir die fase van die frekwensieweergawe $\angle H(e^{j\omega}$. Skets hierdie faseweergawe oor die interval $0 \leq \omega \leq 2\pi$. Wys en motiveer u bewerkings duidelik. (4)

c) Determine a real and closed-form expression for the phase of the frequency response $\angle H(e^{j\omega}$. Sketch this phase over the interval $0 \leq \omega \leq 2\pi$. Show and motivate your working clearly. (4)

Vraag 6 totaal: 10 punte.

Question 6 total: 10 marks.

Vraag 7 Ontwerp 'n eerste-orde keep (*notch*) filter (d.w.s. 'n keep filter met 'n enkele pool en 'n enkele zero) waarmee GS (d.w.s. $\omega = 0$ radiale/monster) volledig verwerp word, wat eenheidsaanwins by $\omega = \pi$ radiale/monster toon en met 'n -3dB afsnyfrekwensie by $\omega = \pi/3$ radiale/monster. Toon u finale ontwerp as 'n direkte-vorm 1 blokdiagram. Wys en motiveer u bewerkings duidelik. (12)

Question 7 Design a first-order notch filter (i.e. a notch filter with a single pole and a single zero) that will completely reject DC (i.e. $\omega = 0$ radians/sample), has unity gain at $\omega = \pi$ radians/sample and its -3dB cutoff at $\omega = \pi/3$ radians/sample. Present your final design as a direct form I block diagram. Show and motivate your working clearly. (12)

Vervolg ...

Continued ...

Vraag 7 (vervolg) *Addisionele ruimte vir berekenings.*

Question 7 (continued) *Additional space for working.*

Vraag 7 totaal: 12 punte.

Question 7 total: 12 marks.

Vraag 8 Die eerste-orde Butterworth hoogdeurlaatfilter gegee deur:

Question 8 The first-order continuous Butterworth high-pass filter given by:

$$H(s) = \frac{s}{s+10}$$

het sy -3dB afsnyfrekwensie by $\Omega = 10$ rad/s. Gebruik hierdie filter as prototipe om met die bilineêre transform 'n diskrete-tyd laagdeurlaatfilter te ontwerp met afsnyfrekwensie $\omega_c = 0.25\pi$ rad/monster. Toon die oordragsfunksie van u finale filter as 'n breuk van polinome in z^{-1}. Toon en motiveer u berekeninge duidelik. *Daar is meer spasie vir u berekeninge op die volgende bladsy.* (9)

has its -3dB cutoff frequency at $\Omega = 10$ rad/s. With this filter as a prototype, use the bilinear transform to design a discrete-time low-pass filter with cutoff frequency $\omega_c = 0.25\pi$ rad/sample. Represent the transfer function of your final filter as a ratio of polynomials in z^{-1}. Show and motivate your calculations. *There is more space for your calculations on the following page.* (9)

Vervolg ...

Continued ...

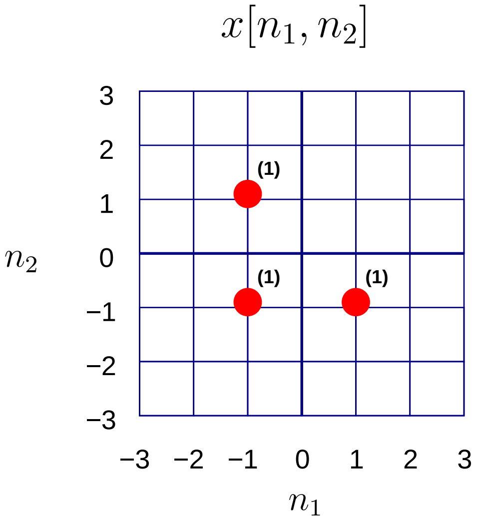
Vraag 8 (vervolg) *Addisionele ruimte vir berekenings*

Question 8 (continued) *Addisional space for working*

Vraag 8 totaal: 9 punte.

Question 8 total: 9 marks.

Vraag 9 Die volgende figuur toon twee tweedimensionlele diskrete seine $x[n_1, n_2]$ en $h[n_1, n_2]$.

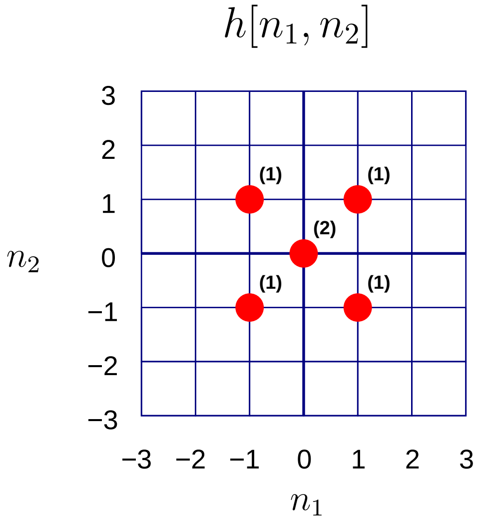


Aanvaar nou dat $h[n_1, n_2]$ die puntverspreidingsfunksie (*point spread function*) is van 'n lineêre skuif-invariante stelsel, waarvan $x[n_1, n_2]$ die intree vorm.

a) Bepaal <u>en skets</u> die respons van die stelsel. Wys en motiveer u bewerkings duidelik. (6)

Vervolg ...

Question 9 The following figure shows two two-dimensional discrete signals $x[n_1, n_2]$ and $h[n_1, n_2]$.



Now assume that $h[n_1, n_2]$ is the point spread function of a linear shift-invariant system, for which $x[n_1, n_2]$ is the input.

a) Determine <u>and sketch</u> the response of the system. Show and motivate your working clearly. (6)

Continued ...

Vraag 9 (vervolg)

b) Sou u $h[n]$ as laag-deurlaat of hoog-deulaat beskryf? Motiveer u antwoord duidelik. (2)

Vraag 9 totaal: 8 punte.

Question 9 (continued)

b) Would you describe $h[n]$ as low-pass or high-pass? Motivate your answer clearly. (2)

Question 9 total: 8 marks.

Vraestel totaal: 90 **Question-paper total: 90**