FAKULTEIT INGENIERSWESE
FACULTY OF ENGINEERING

Stelsels en Seine 414 ***Systems and Signals 414***			**Eerste Eksamen** **31 Mei 2012** ***First Examination*** ***31 May 2012***
Tydsduur ***Duration***	**3 h**	**Volpunte** ***Full marks***	**90**
Eksaminator: ***Examiner:***			**Mede-eksaminator(e):** ***Co-examiner(s):***
T.R. Niesler			**J.A. du Preez**

Sertifisering
Certification

Opgestel: Set: Eksaminator Examiner	Gemodereer: Moderated: Mede-eksaminator Co-examiner

Kandidaatinligting
Candidate's particulars

Van: ***Surname:***	
Voorname: ***First Names:***	
Studentenommer: ***Student number:***	
Handtekening: ***Signature:***	

Lees asseblief noukeurig die instruksies op die volgende bladsy.
Please read instructions on the next page carefully.

INSTRUKSIES

- ***Vul u naam en studentenommer in soos aangedui op die voorblad van hierdie vraestel !***
- Lees die inligting op beide hierdie vraestel en die meegaande eksamenboek. Verskaf u gegewens op beide.
- Gee u antwoorde op die beskikbare plek onderaan elke vraag *op die vraestel*. **Die meegaande eksamenboek is beskikbaar net vir rofwerk en word nie gemerk nie.**
- U mag u voorgeskrewe handboek, Proakis & Manolakis sowel as die klasnotas soos in die lesings uitgedeel raadpleeg. Normale notas/kommentaar daarin is in orde. Geen verdere notas (ook nie in 'n sakrekenaar) word toegelaat nie.
- Toon en motiveer u redenasies altyd volledig. ***Punte sal afgetrek word indien dit nie gedoen word nie***. Omskryf in woorde wat u probeer doen - dit tel in u guns indien u nie 'n berekening suksesvol deurvoer nie.
- Waar gegewens na u mening ontbreek, maak sinvolle, gemotiveerde aannames.
- Skryf met 'n pen. Sketse kan egter in potlood gemaak word.
- Plaas die voltooide vraestel in die rofwerkboek en handig beide (volledig) in.

INSTRUCTIONS

- ***Fill in your name and student number in the space provided on the cover of this question paper!***
- Read the information on this question paper and on the accompanying examination book. Provide your details on both.
- Provide your answers in the space allocated after each question *on this question paper*. **The accompanying examination book is for rough-work only and will not be marked.**
- You may consult the prescribed handbook, Proakis & Manolakis as well as the handouts given in class. Normal notes/comments in it are acceptable. All further notes (also in a calculator) are forbidden.
- Always show and motivate your reasoning fully. ***Marks will be deducted for failing to do so***. Describe what you are trying to do - this counts in your favour with unsuccessful calculations.
- If in your opinion any information is missing, make reasonable, motivated assumptions.
- Write with a pen. Sketches may be in pencil.
- Put the completed question paper inside the rough-work book and hand both (everything) in.

FOR MARKING PURPOSES ONLY

Question	1	2	3	4	5	6	7	8	9	10	11	12	**Tot**
Mark													
Check													

TOTAL:

Vraag 1 Beskou die volgende kontinue-tyd sein $x(t)$, met t die tyd in sekondes:

Question 1 Consider the following continuous-time signal $x(t)$, where t is the time in seconds:

$$x(t) \quad = \quad 1.2 - \cos(300\pi t) + \sin(300\pi t) - 2\sin(500\pi t) + \cos(700\pi t + \frac{\pi}{4})$$

Die diskrete-tyd sein $x[n]$ word verkry deur $x(t)$ te monster sodat $x[n] = x(nT)$ met $T = 0.001s$, die monsterperiode in sekondes. Skets die 20-punt drywingsdigtheidspektrum $S_{xx}[k]$ van die eerste 20 monsters van $x[n]$, d.w.s.: $x[0], x[1], \ldots x[19]$. Dui asse, amplitudes en frekwensies deeglik aan. (8)

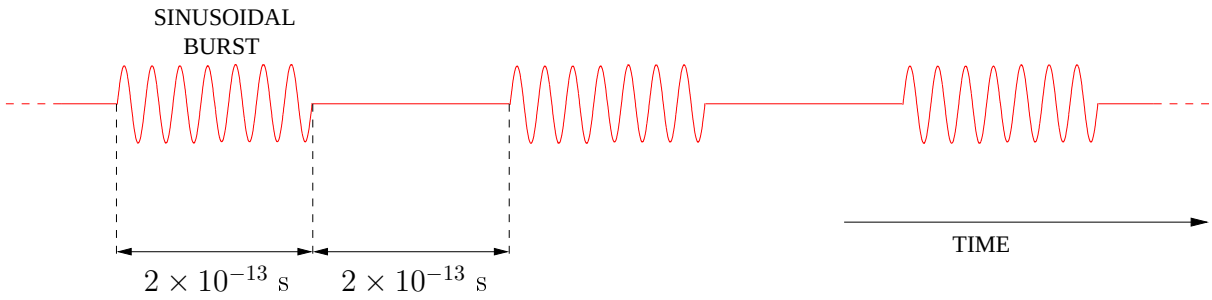
The discrete-time signal $x[n]$ is obtained by sampling $x(t)$ so that $x[n] = x(nT)$ with $T = 0.001s$ the sampling period in seconds. Sketch the 20-point power density spectrum $S_{xx}[k]$ using the first 20 samples of $x[n]$, i.e.: $x[0], x[1], \ldots x[19]$. Label axes, amplitudes and frequencies thorougly. (8)

Vraag 1 totaal: 8 punte.

Question 1 total: 8 marks.

Vraag 2 Aan boord van die ruimteskip *Enterprise* onderskep Luitenant Uhara 'n dowwe inkomende boodskapsein. Met behulp van die ruimteskip se rekenaar analiseer Kommandeur Spock die sein en bepaal dat dit bestaan uit reëlmatig-gespasieerde kort sinusvormige pulse (lengte 2×10^{-13}s) in die byna-infrarooi gebied van die elektromagnetiese spektrum. Die sein word in onderstaande figuur geïllustreer.

Question 2 On the bridge of the starship *Enterprise*, Lieutenant Uhura detects a faint incoming signal. Commander Spock analyses it with the ship's computer and finds that it consists of regular short sinusoidal bursts (length 2×10^{-13}s) in the near infrared part of the electromagnetic spectrum, as illustrated in the following figure.



Die sinusvormige pulse het twee nabygeleë maar afsonderlike frekwensies: 2×10^{14}Hz en 2.05×10^{14}Hz, onderskeidelik. Die informasie wat in die boodskapsein bevat word kan ontsyfer word deur die frekwensie van elke sinusvormige puls te identifiseer. Deur 'n dilitiumkristal-kragbron te gebruik, verbeter Kommandeur Spock die analoog-na-syfer omsetter (ADC) wat deur die Enterprise se wyeband-skandeerder gebruik word om te werk teen 'n monsterfrekwensie van twee duisend Terraherz (d.w.s. 2000×10^{12}Hz). Aanvaar dat Spock 'n Hamming venster en 'n DFT/FFT in sy berekeninge gebruik en bepaal of die *Enterprise* wel die vreemde sein sal kan ontsyfer. Toon en motiveer al u berekeninge volledig, op 'n standaard aanvaarbaar vir Starfleet. Toon en motiveer u berekeninge. (8)

The sinusoidal bursts have two closely-spaced but distinct frequencies: 2×10^{14}Hz and 2.05×10^{14}Hz respectively. The information encoded in the signal can be decoded by identifying the frequency of each burst. By improvising a secondary dilithium crystal power source, Commander Spock has improved the analog-to-digital converter (ADC) used by the *Enterprise's* wideband scanner to operate at a sampling frequency of two thousand Terraherz (i.e. 2000×10^{12}Hz). Assuming that Spock uses a Hamming window and the DFT/FFT in his calculations, determine whether the *Enterprise* can reliably decode the alien message. Show and motivate all calculations fully and to Starfleet standards. (8)

Vraag 2 (vervolg) *Addisionele ruimte vir berekenings.*

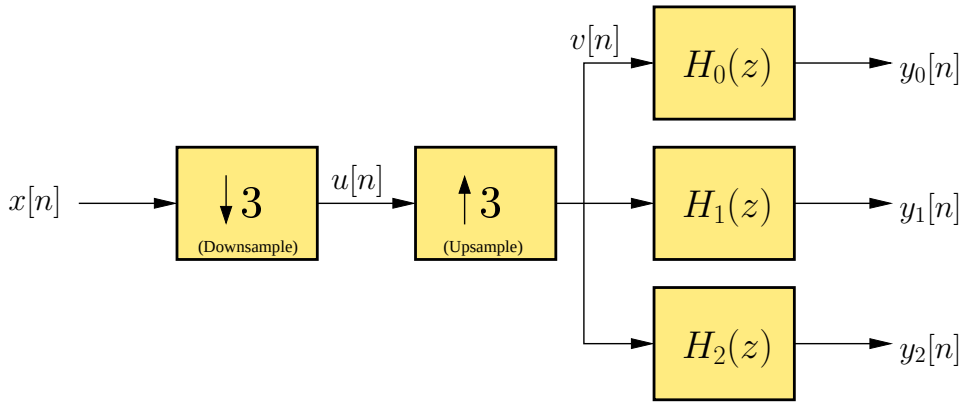
Question 2 (continued) *Additional space for working.*

Vraag 2 totaal: 8 punte.

Question 2 total: 8 marks.

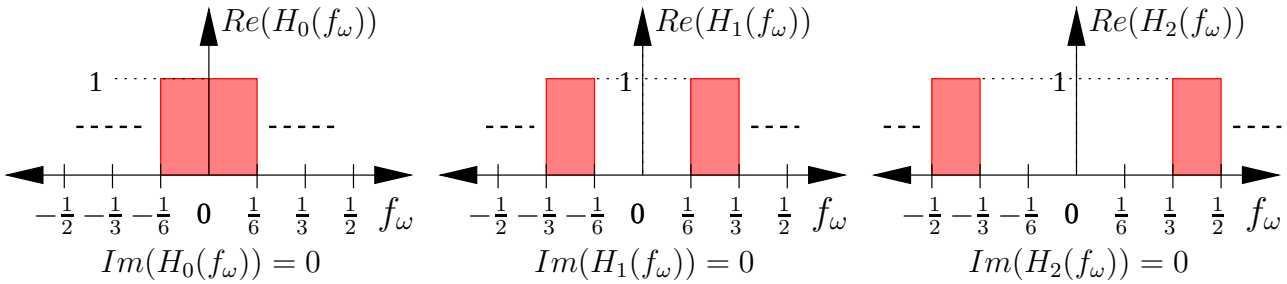
Vraag 3 Beskou die volgende blokdiagram van 'n digitale seinprossesserings stelsel.

Question 3 Consider the following block diagram of a digital signal processing system.



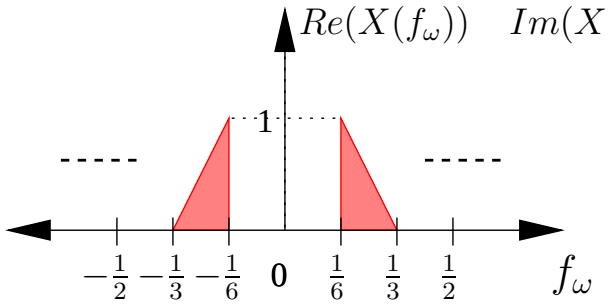
Die intreesein $x[n]$ word met 'n faktor 3 eers afgemonster (downsample) en dan met dieselvde faktor weer opgemonster (upsample). Die resulterende sein word dan gefilter deur drie lineêre tyd-invariante stelsels met oordragsfunksie $H_0(z)$, $H_1(z)$ en $H_2(z)$ onderskeidelik, soos in die volgende skets aangedui.

An input signal $x[n]$ is downsampled by a factor 3 and subsequently upsampled by the same factor. Next, the resulting signal is filtered by three linear time-invariant systems with transfer functions $H_0(z)$, $H_1(z)$ and $H_2(z)$ respectively, as shown in the following sketch.



Die intreesein $x[n]$ is reël met die volgende Fourier transform $X(f_\omega)$.

The input signal $x[n]$ is real and has the following Fourier transform $X(f_\omega)$.



Bepaal en skets die spektra $Y_0(f_\omega)$, $Y_1(f_\omega)$ en $Y_2(f_\omega)$ van die uittreeseine seine $y_0[n]$, $y_1[n]$ en $y_2[n]$ onderskeidelik oor die interval $-1 \leq f\omega \leq 1$ siklusse/monster. Toon en motiveer u berekeninge. Dui asse, amplitudes en frekwensies deeglik aan. (10)

Determine and sketch the spectra $Y_0(f_\omega)$, $Y_1(f_\omega)$ and $Y_2(f_\omega)$ of the output signals $y_0[n]$, $y_1[n]$ en $y_2[n]$ respectively over the interval $-1 \leq f\omega \leq 1$ cycles/sample. Show and motivate your calculations. Label axes, amplitudes and frequencies thorougly. (10)

Vervolg ... Continued ...

Vraag 3 (vervolg) *Addisionele ruimte vir berekenings.*

Question 3 (continued) *Additional space for working.*

Vraag 3 totaal: 10 punte.

Question 3 total: 10 marks.

Vraag 4 'n Lineêre tyd-invariante (LTI) stelsel met impulsweergawe $h_1[n]$ word as omkeerbaar beskou as daar 'n tweede LTI stelsel met impulsweergawe $h_2[n]$ bestaan sodat $h_1[n] * h_2[n] = \delta[n]$, waar '*' die diskrete konvolusie en $\delta[n]$ die diskrete impulsfunksie aandui. Beskou nou die volgende vier pare van impulsweergawes $h_1[n]$ en $h_2[n]$. Die funksie $u[n]$ dui die diskrete eenheidstrap aan.

(a) $h_1[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2]$
(b) $h_1[n] = u[-n-1]$
(c) $h_1[n] = 0.5^n u[n]$
(d) $h_1[n] = 10\delta[n] - 3\delta[n-1] - 4\delta[n-2]$

Bepaal in elke geval of $h_2[n]$ 'n inverse stelsel is vir $h_1[n]$ of nie. Wys en motiveer u bewerkings duidelik. (10)

Question 4 A linear time-invariant (LTI) system with impulse response $h_1[n]$ is considered to be invertible when a second LTI system with impulse response $h_2[n]$ exists such that $h_1[n] * h_2[n] = \delta[n]$, where '*' denotes the discrete convolution and $\delta[n]$ is the discrete impulse function. Now consider the following four pairs of impulse responses $h_1[n]$ and $h_2[n]$. The function $u[n]$ denotes the discrete unit step.

$h_2[n] = \frac{1}{3}\delta[n] + \frac{1}{2}\delta[n-2] + \delta[n-3]$
$h_2[n] = \delta[n-1] - \delta[n]$
$h_2[n] = \delta[n] - 0.5\delta[n-1]$
$h_2[n] = [2(0.8^n) + 3(0.4^n)] \cdot u[n]$

Determine in each case whether or not $h_2[n]$ is an inverse system for $h_1[n]$. Show and motivate your working clearly. (10)

Vraag 4 totaal: 10 punte.

Question 4 total: 10 marks.

Vraag 5 Beskou die volgende diskrete-tyd sein $x[n]$.

Question 5 Consider the following discrete-time signal $x[n]$.

$$x[n] = a^n u[n]$$

a) Die funksie $u[n]$ dui die diskrete eenheidstrap aan. Bepaal en skets die konvergensiegebied (*region of convergence, ROC*) van $R_{xx}(z)$ in terme van a, waar $R_{xx}(z)$ die z-transform van $r_{xx}[n]$ is, en $r_{xx}[n]$ die outokorrelasie van $x[n]$ aandui. Toon en motiveer u berekeninge. (6)

a) The function $u[n]$ denotes the discrete unit step. Determine and sketch the region of convergence (ROC) of $R_{xx}(z)$ in terms of a, where $R_{xx}(z)$ is the z-transform of $r_{xx}[n]$, and $r_{xx}[n]$ is the autocorrelation of $x[n]$. Show and motivate your working clearly. (6)

b) Bepaal nou die bestaansvoorwaardes, in terme van a, vir $R_{xx}(z)$. Motiveer u antwoord. (2)

b) Now determine, in terms of a, the conditions for the existence of $R_{xx}(z)$. Motivate your answer. (2)

Vraag 5 totaal: 8 punte.

Question 5 total: 8 marks.

Vraag 6 Beskou die volgende z-transform:

$$X(z) = \frac{4 + 0.5z^{-1} - 0.5z^{-3}}{1 - 0.5z^{-1}}$$

Bepaal 'n geslote-vorm uitdrukking vir die kousale diskrete-tyd sein $x[n]$ waarvan $X(z)$ die z-transform is. Skets u antwoord, en dui spesifiek die eerste 5 waardes daarvan aan ($n = 0, 1, 2, 3, 4$). (8)

Question 6 Consider the following z-transform:

Determine a closed-form expression for the causal discrete-time signal $x[n]$ of which $X(z)$ is the z-transform. Sketch your answer, specifically indicating the first 5 values thereof ($n = 0, 1, 2, 3, 4$). (8)

Vraag 6 totaal: 8 punte.

Question 6 total: 8 marks.

Vraag 7 Beskou die diskrete-tyd stelsel met intree $x[n]$ en uittree $y[n]$, met monsterfrekwensie van $8000Hz$, wat deur die volgende lineêre konstante-koëffisiënt verskilvergelyking beskryf word:

Question 7 Consider the discrete-time system with input $x[n]$ and output $y[n]$ which operates at a sampling rate of $8000Hz$, and is described by the following linear constant-coefficient difference equation :

$$y[n] = x[n] + y[n-1]$$

Bepaal 'n vereenvoudigde reële uitdrukking vir die amplitude en fase van die frekwensieweergawe $H(e^{j\omega})$. Skets beide oor die interval $0 \leq \omega \leq 2\pi$ en dui duidelik die amplitudes en fases by $\omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$ aan. (12)

Determine a simplified real expression for the magnitude and phase of the frequency response $H(e^{j\omega})$. Sketch both over the interval $0 \leq \omega \leq 2\pi$ and indicate the amplitudes and phases at $\omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$ (12)

Vraag 7 (vervolg) *Addisionele ruimte vir berekenings.*

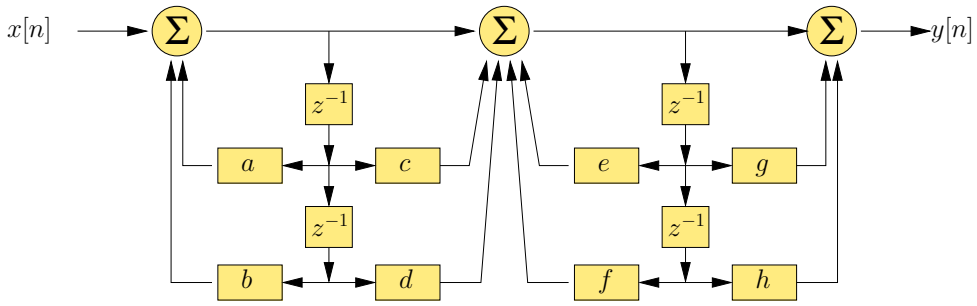
Question 7 (continued) *Additional space for working.*

Vraag 7 totaal: 12 punte.

Question 7 total: 12 marks.

Vraag 8 Beskou die volgende blokdiagram van 'n lineêre tyd-invariante (LTI) stelsel.

Question 8 Consider the following block diagram if a linear time-invariant (LTI) system.



Die stelsel in die blokdiagram het die volgende oordragsfunksie $H(z)$.

The system in the block diagram has the following transfer function $H(z)$.

$$H(z) = \frac{(1+z^{-1})^4}{(1-2z^{-1}+\frac{7}{8}z^{-2})(1+z^{-1}+\frac{1}{2}z^{-2})}$$

a) Bepaal waardes vir die koëffisiënte a tot h. Motiveer u antwoord. (4)

a) Determine values for the coefficients a to h. Motivate your answer. (4)

b) Is u oplossing vir die vorige vraag uniek? Motiveer u antwoord. (2)

b) Is your solution to the previous question unique? Motivate your answer. (2)

Vraag 8 totaal: 6 punte.

Question 8 total: 6 marks.

Vraag 9 Beskou die diskrete-tyd stelsel wat deur die volgende lineêre konstante-koëffisiënt verskilvergelyking beskryf word:

$$y[n] - 1.2y[n-1] + 0.32y[n-2] = 0$$

Bepaal 'n geslote-vorm uitdrukking vir die weergawe $y[n]$ van die stelsel, met $n \geq 0$, gegee die volgende begintoestande.

$$\begin{aligned} y[-1] &= 1 \\ y[-2] &= 1 \end{aligned}$$

Skets u resultaat, en dui spesifiek die waardes van $y[0]$, $y[1]$ en $\lim_{n\to\infty} y[n]$ aan. Wys en motiveer u bewerkings duidelik. (11)

Question 9 Consider the discrete-time system described by the following linear constant-coefficient difference equation :

$$y[n] - 1.2y[n-1] + 0.32y[n-2] = 0$$

Determine a closed-form expression for the response $y[n]$ of the system, with $n \geq 0$, given the following intial condidtions.

$$\begin{aligned} y[-1] &= 1 \\ y[-2] &= 1 \end{aligned}$$

Sketch your result, indicating specifically the values of $y[0]$, $y[1]$ and $\lim_{n\to\infty} y[n]$. Show and motivate your working clearly. (11)

Vraag 9 (vervolg) *Addisionele ruimte vir berekenings.*

Question 9 (continued) *Additional space for working.*

Vraag 9 totaal: 11 punte.

Question 9 total: 11 marks.

Vraag 10 Die tweede-orde Butterworth laagdeurlaatfilter gegee deur:

$$H(s) = \frac{394784}{s^2 + 889s + 394784}$$

het sy afsnyfrekwensie by ongeveer 100Hz. Gebruik hierdie filter as prototipe om met die bilineêre transform 'n diskrete-tyd laagdeurlaatfilter te ontwerp met afsnyfrekwensie $\omega_c = 0.35\pi$ rad/monster. Skets 'n direkte-vorm I blokdiagram van u antwoord. (9)

Question 10 The second-order continuous Butterworth low-pass filter given by:

has its cutoff frequency at approximately 100Hz. With this filter as a prototype, use the bilinear transform to design a discrete-time low-pass filter with cutoff frequency $\omega_c = 0.35\pi$ rad/sample. Sketch a direct-form I block diagram of your result. (9)

Vervolg ... Continued ...

Vraag 10 (vervolg) *Addisionele ruimte vir berekenings*

Question 10 (continued) *Additional space for working*

Vraag 10 totaal: 9 punte.

Question 10 total: 9 marks.

Vraestel totaal: 90 **Question-paper total: 90**