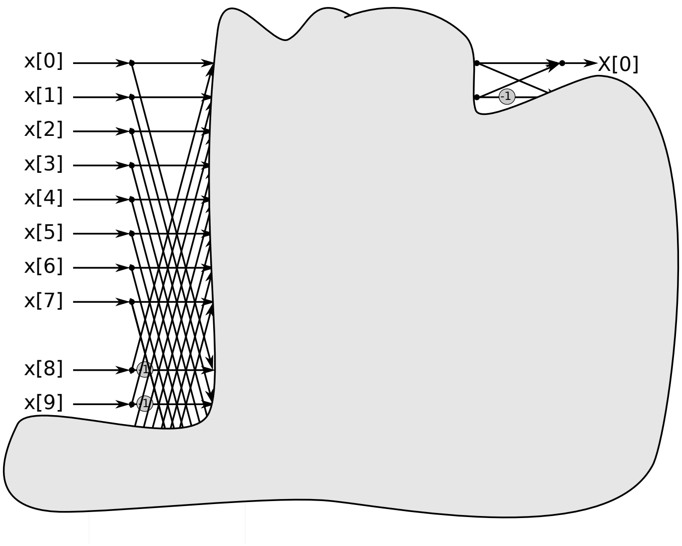
Note: because of space issues, this tut will take place in E202.

Question 1

Complete the diagram for the 16-point decimation-in-frequency FFT of $x[n]$. (Don't worry too much about how pretty your diagram looks):



Check yourself: Output order of X[k]: $X[0], X[8], X[4], X[2], \ldots$

Question 2

Consider the following DFT/IDFT pairs. (The ↑ shows the position of the 0 index):

$$\underset{\uparrow}{[1}, 1, 1, 1] \underset{\text{IDFT}}{\overset{\text{DFT}}{\rightleftarrows}} \underset{\uparrow}{[4}, 0, 0, 0]$$

$$[\underset{\uparrow}{0 + j1},\ 1 + j1,\ 2 + j1,\ 3 + j1] \underset{\text{IDFT}}{\overset{\text{DFT}}{\rightleftarrows}} [\underset{\uparrow}{6 + j4}, -2 + j2,\ -2 + j0,\ -2 - j2]$$

Use these DFT pairs and from that determine the DFT $P[k]$ for

$$p[n] = [\underset{\uparrow}{0}, 1, 2, 3]$$

Check yourself: Slide 1.72 discusses $x[n] = x_1[n] + jx_2[n]$

Question 3

From the definition of the discrete convolution, show that discrete convolution is distributive over addition, i.e. that:

$$x[n] * [h_1[n] + h_2[n]] = x[n] * h_1[n] + x[n] * h_2[n],$$

where $x[n]$, $h_1[n]$, and $h_2[n]$ are all energy signals. Show and motivate your working clearly.

Question 4

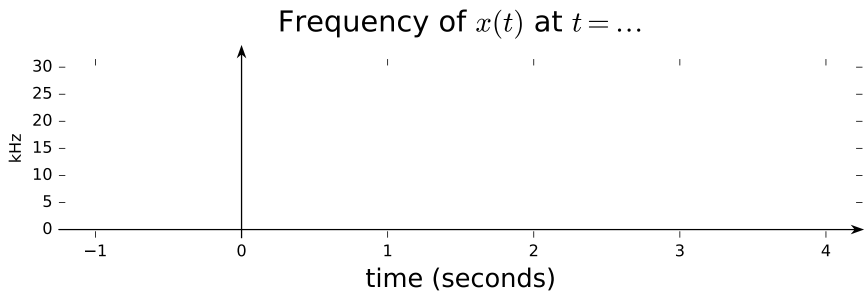
If you still do not understand the purpose of the spectrogram, go watch youtube.com/watch?v=vvr9AMWEU-c to get an idea of what we mean by instantaneous frequency over time.

A discrete-time signal $x[n]$ is obtained by sampling a continuous-time signal $x(t)$ at a sampling frequency of 40kHz. The signal $x(t)$ consists of a single sinusoid, increasing linearly in frequency from 0Hz to 20kHz in the space of 2 seconds, then decreases back to 0Hz within the same space of time, and repeating this process.

The signal $x(t)$ is analysed using a spectrogram with the aim of determining the instantaneous frequency every 2ms to within a frequency resolution of 200Hz, with an assurance of full main lobe separation. We assume the use of a Hamming window.

a) Draw a rough sketch of the magnitude of $X(f)$, the spectrum (FFT) of $x(t)$.

b) Draw a sketch of the instantaneous frequency of $x(t)$ over time on the following axis:



c) Analysis occurs using a spectrogram in which the frame length is $N = 32$ samples, the frame skip is $R = 16$ samples, and a Hamming window is applied to every frame. Are these appropriate values for the analysis? Explain why (or why not). Show and motivate clearly.

Check yourself: (1) Find the Δt for $\Delta 200Hz$ frequency change in seconds and convert this to samples to find upper bound for N. (2) Find the $B\omega$ for full lobe separation for Δ200Hz - the lower bound for N. (3) Find the lower bound for the framerate R.

d) Analysis now occurs using a spectrogram in which the frame length is $N = 2048$ samples, the frame skip is $R = 256$ samples, and a Hamming window is applied to every frame. Are these appropriate values for the analysis? Explain why (or why not). Show and motivate clearly.

e) Assuming that a Hamming window is applied to every frame, determine ideal values for N and R.

Bonus Question

Team Jacob or team Edward? Motivate your answer.

Additional Question 1

Use the DFT pairs from Question 2 (along with the answer) and from that

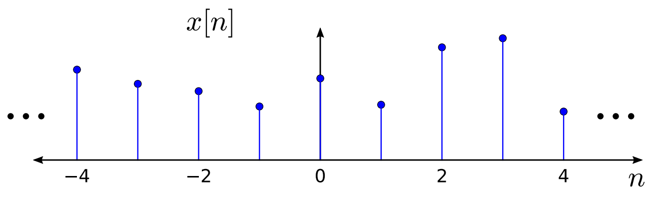
determine the DFT $Q[k]$ for $\quad q[n] = [\underset{\uparrow}{0}, 1, 1, 1, 2, 1, 3, 1],$

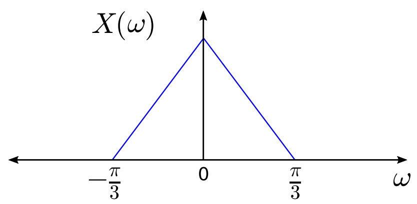
and the IDFT $r[n]$ for $\quad R[k] = [1 \underset{\uparrow}{+} j0,\ 1 + j1,\ 1 + j2,\ 1 + j3].$

Check yourself: From question 2: $p[n] = [6 + j0,\ -2 + j2,\ -2 + j0,\ -2 - j2]$. Use the decimation in time algorithm and split $q[n]$ into even and odd indices. $Q[k] = [10, -2 + j2, \ldots, -2 - j2]$. For $r[n]$ see slide 1.73.

Additional Question 2

Consider the following discrete-time signal $x[n]$ with Fourier transform $X(\omega)$:





a) If signal x_s is taken as follows:

$$x_s[n] = \begin{cases} x[n], & \text{for } n = 0, \pm 2, \pm 4, \ldots \\ 0, & \text{for } n = \pm 1, \pm 3, \pm 5 \ldots \end{cases}$$

Compute and sketch the Fourier transform $X_s(\omega)$. Can we reconstruct $x[n]$ from $x_s[n]$? How, or why not?

b) Signal x_d is taken by decimating $x[n]$ by a factor of D=2:

$$x_d[n] = x[2n], \qquad \text{for all } n$$

Sketch the Fourier transform $X_d(\omega)$. Can we reconstruct $x[n]$ from $x_s[n]$? How, or why not?