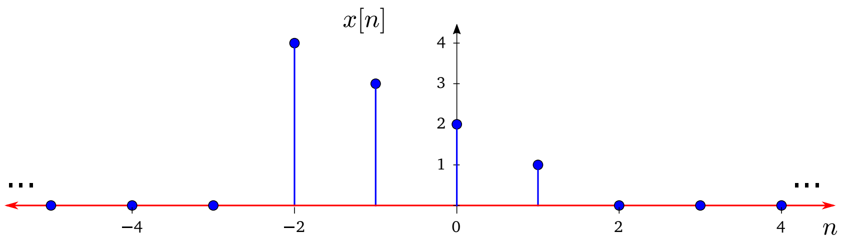
Question 1

Consider the following 4-sample discrete-time signal $x[n]$:

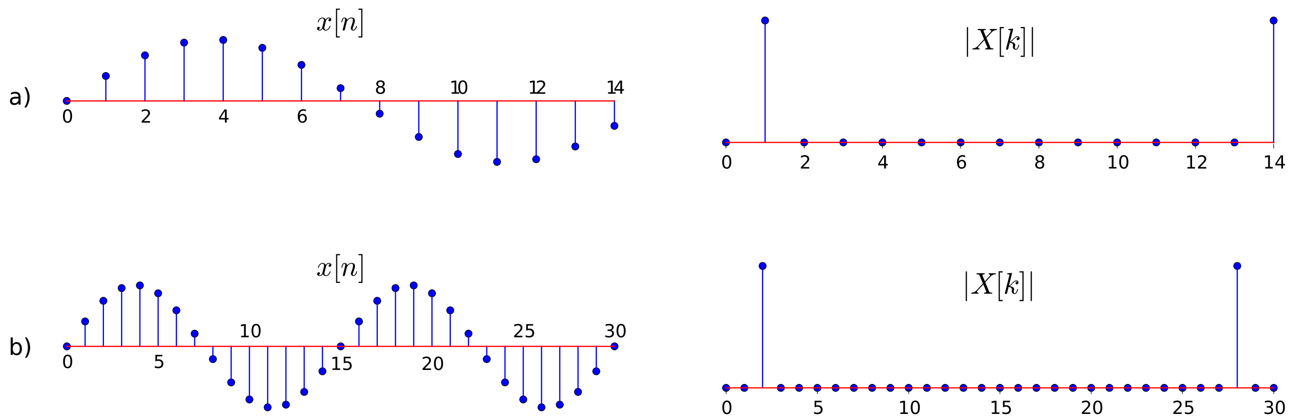


Now determine $y[n] = x[3 - n] + x[n - 8]$, and present your answer as a sketch.

(Check yourself: $y[5] = 4$ and $y[6] = 4$)

Question 2

Indicate if the discrete Fourier transformation $X[k]$ is in fact the transformation of the discrete time signal $x[n]$ for both case (a) with 15 samples and case (b) with 31 samples. Write down either “true” or “false”, and provide at least two reasons for any falsehood. The cheat-sheet provided on the website might be of some help: http://courses.ee.sun.ac.za/Systems_and_Signals_414/content/cheatsheet.pdf

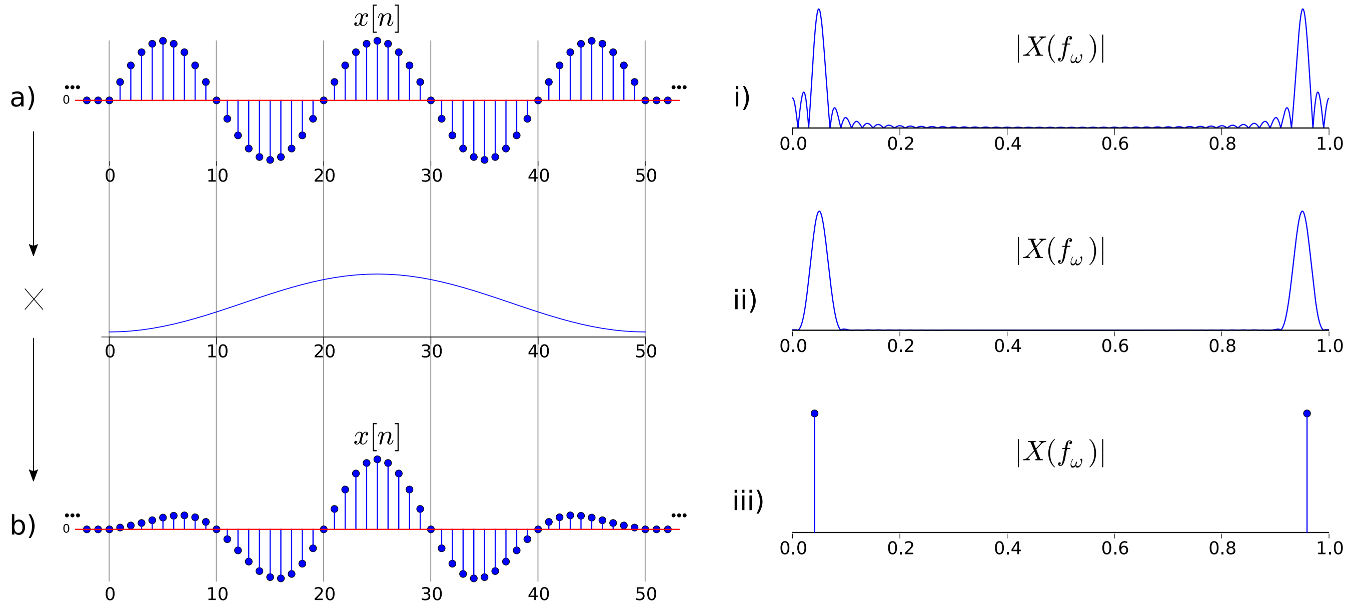


(Check yourself: Would repeating (a) into periods result in a clean sinosoid? Would repeating (b) into periods result in a clean sinosiod? If not how would the DFT react? Is impulses in the DFT placed at the correct samples?)

Question 3

Refer to the following image and your notes regarding windowing. On the left hand side we have two discrete-time signals, indicated as (a) and (b). The signal at (b) is merely a Hamming-windowed version of the signal at (a), as illustrated in the sketch. On the right hand side we provided three possible Fourier transformations (i), (ii) and (iii) such that $x[n] \leftrightarrow X(f_\omega)$.

For the signal at (a), which one of the (i), (ii), or (iii) signals does it correspond with? State your reason. Now repeat the question, but with regards to (b).



(Check yourself: The (i) belongs to (a), the (ii) belongs to (b))

Question 4

Consider the following finite-length discrete time signal $x[n]$:

$$x[n] = \begin{cases} e^{j\omega_o n} & \text{for } 0 \leq n \leq N-1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine the discrete-time Fourier transform (DTFT) $X(\omega)$ of $x[n]$ by directly using the definition of the DTFT (Slide 1.24). Simplify your answer to a ratio of polynomials of $e^{j\omega}$ (and no further).

(Check yourself: $(e^{j(\omega-\omega_o)N} - 1)/(\square)$. Look for something like this: (1) $X(\omega) = \square$; (2) $e^{\square} X(\omega) = \square$; (3): $X(\omega) - e^{\square} X(\omega) = \square$. Note that $\square$ is just a lazy substitute for any different stuff.)

(b) Determine the N-point discrete Fourier transform (DFT) $X[k]$ of $x[n]$. See Slide 1.46 "DISCRETISE IN FREQUENCY".

(Check yourself: $X[k] = X(\omega)|_{\omega=2\pi k/N}$)

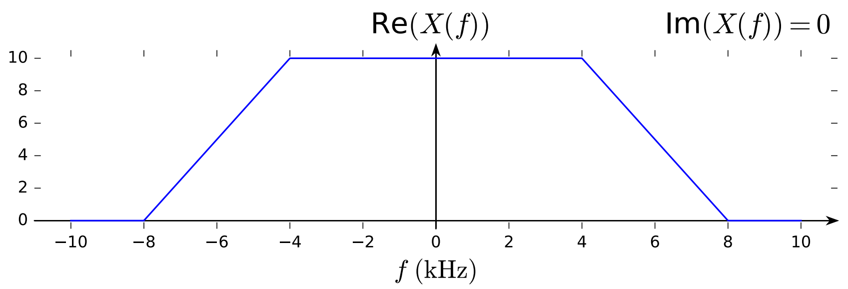
(c) Using only the DTFT example pairs (Slide 1.29) and the properties of the DTFT (Slide 1.28), determine the DTFT $X(\omega)$ of $x[n]$.

(Check yourself: See it as a time shifted rectangular-window of size $2L+1$ which is multiplied by $e^{j\omega_o n}$)

(d) **Homework:** You should have a slightly different form for $X(\omega)$ between (a) and (c). Plot the magnitude of the results obtained in (a) and (c) using Python, with $\omega_o = \pi/5$ and $N = 40$, in order to verify your results.

Question 5

Consider the bandlimited continuous-time signal $x(t)$ that has the following spectrum (Fourier transform) $X(f)$.



A discrete-time signal $x[t]$ is obtained by sampling the continuous-time signal $x(t)$ at a sampling frequency $f_s =$ 10kHz. Sketch the spectrum $X(f_\omega)$ of the sampled signal $x[n]$ over the interval $-1 < f_\omega < 1$, where f_ω is the frequency in cycles/sample. Label axes, amplitudes and frequencies thoroughly. Show and motivate your calculations.

(Check yourself: $X(0) = 100k$, $X(0.5) = 150k$, $X(0.6) = 150k$, $X(0.8) = 100k$)

Additional questions

Additional Question 1

Consider the following discrete-time signal $x[n]$.

$$x[n] = \begin{cases} 1 & \text{for } n = -2, n = 0, \text{ and } n = 2 \\ 0 & \text{elsewhere} \end{cases}$$

Determine a real close-form expression for the Fourier transform $X(\omega)$ of this signal, where ω is the frequency in radians/sample. Then sketch the magnitude and phase of your answer over the interval $-2\pi \leq \omega \leq 2\pi$. Label axes, amplitudes and frequencies thoroughly. Show and motivate your calculations.

(Check yourself: $H(\omega) = 1 + 2\cos(2\omega)$. Draw $|H(\omega)|$ with an anti-symmetric phase)

Additional Question 2

Draw sketches of $x[n]$ as 2D graphs, and draw sketches of $X[k]$, the DFT of $x[n]$, as 3D graphs (including the real and imaginary axis as shown below), for the following $x[n]$:

(a) $x[n] = \sin(\frac{1}{8} \cdot 2\pi n)$ for $n = \{0, 1, \cdots, 39\}$

(Check yourself: $x[n]$ completes 5 periods, $X[5] = -20j$, and $X[35] = 20j$

(b) $x[n] = \sin(\frac{1}{8} \cdot 2\pi n)$ for $n = \{0, 1, \cdots, 7\}$

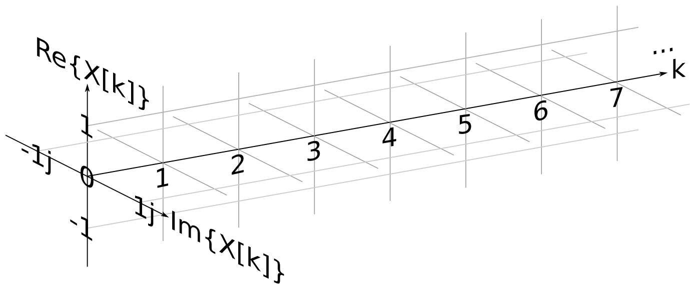
(Check yourself: $x[n]$ completes 1 period, $X[1] = -4j$, and $X[7] = 4j$

(c) $x[n] \leftarrow$ sampled from $x(t) = \sin(250 \cdot 2\pi t)$ at $f_s = 2k\text{Hz}$ for 40 samples.

(Check yourself: same as (a))

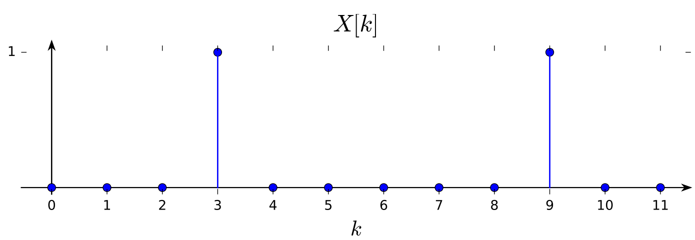
(d) $x[n] \leftarrow$ sampled from $x(t) = \sin(250 \cdot 2\pi(t - 0.001))$ at $f_s = 2k\text{Hz}$ for 40 samples.

(Check yourself: time delay results in a phase shift in the freq. domain)



Additional Question 3

Consider $X[k] = \{0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0\}$ and $x[n]$ as the IDFT of $X[k]$.



(a) Draw a rough sketch of $|Y[k]|$ where $y[n]$ is $x[n]$ zero-padded to be exactly four times the length of $x[n]$.

(Check yourself: Total 48 points, main-lobes centred at sample 12 and sample 36 with bandwidth of 8 samples, each side-lobe has a bandwidth of 4 samples)

(b) Draw a rough sketch of $|Q[k]|$ where $q[n]$ is a hamming window of length $x[n]$ applied to $x[n]$ then zero-padded to be exactly four times the length of $x[n]$.

(Check yourself: Total 48 points, main-lobes centred at sample 12 and sample 36 with bandwidth of 16 samples, each side-lobe has a bandwidth of 4 samples)

(c) **Homework:** Verify your findings by generating Python plots for $|Y[k]|$ and $|Q[k]|$ by using np.hamming, np.fft.fft, np.fft.ifft, and concatenation directly.