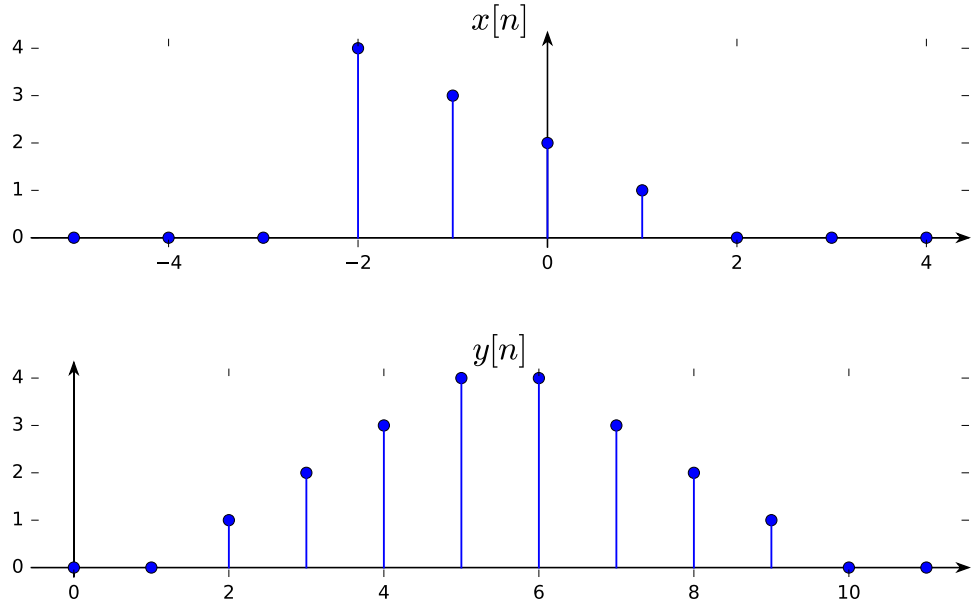
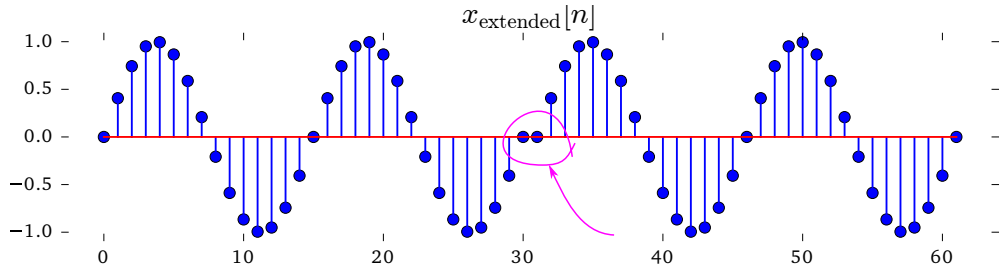
MEMO

Question 1

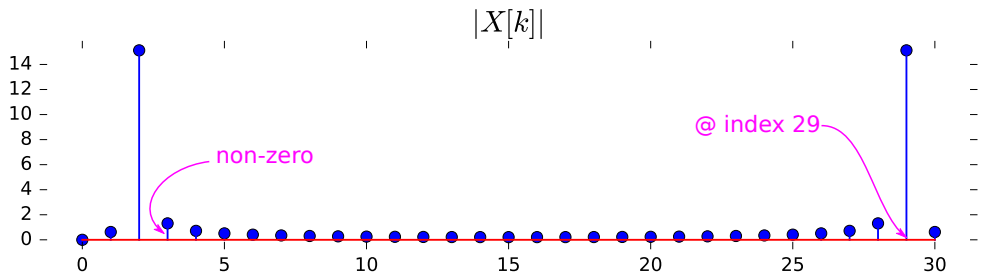


Question 2

(a): True: A sampled sinusoid with 15 samples depicting exactly one period will have this DFT.
(b): False: A sampled sinusoid with 31 samples depicting one period plus an extra annoying little sample is not a smooth sinusoid any more. For example, extending to two periods will show the following "glitch" at sample 30 and 31:



The DFT will thus have non-zero elements near the peaks. The peaks at index 28 should also in reality be at index 29. Here is the true DFT:



Question 3

(a)↔(i) and (b)↔(ii)
Note that (a)'s period is not infinitely repeating and therefore the DTFT will not show as single impulses. Also, the signal (a) is very sharply windowed with a rectangular-window and this type of window will have a small lobe-width , but to the avail of side-lobes with high magnitude. This can be seen in (i).
The signal in (b) is windowed with a Hamming-window, this will result in side-lobes of lower magnitude, but to the avail of larger lobe-width (the main-lobe occupies more bandwidth). This is seen in (ii).

Question 4

(a) $x[n] = e^{j\omega_0 n}$ for $n = \{0, 1, \ldots, N-1\}$

$$X(\omega) = \sum_{n=0}^{N-1} e^{j\omega_0 n} e^{-j\omega n} = \sum_{n=0}^{N-1} e^{-j(\omega-\omega_0)n}$$

$$X(\omega) = e^{-j(\omega-\omega_0)\cdot 0} + e^{-j(\omega-\omega_0)1} + \ldots + e^{-j(\omega-\omega_0)(N-1)}$$

$$e^{-j(\omega-\omega_0)} X(\omega) = e^{-j(\omega-\omega_0)1} + \ldots + e^{-j(\omega-\omega_0)(N-1)} + e^{-j(\omega-\omega_0)N}$$

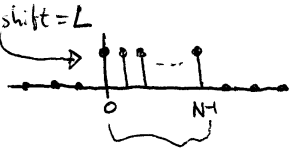
$$X(\omega) - e^{-j(\omega-\omega_0)} X(\omega) = 1 - e^{-j(\omega-\omega_0)N}$$

$$\therefore X(\omega) = \frac{1 - e^{-j(\omega-\omega_0)N}}{1 - e^{-j(\omega-\omega_0)}}$$

Alternative: $\dfrac{e^{j(\omega-\omega_0)N} - 1}{e^{j(\omega-\omega_0)} - e^{j(N-1)(\omega-\omega_0)}}$

(b) $\omega \longrightarrow \frac{2\pi}{N} k$

$$X[k] = \frac{1 - e^{-j\left(\frac{2\pi}{N}k - \omega_0\right)N}}{1 - e^{-j\left(\frac{2\pi}{N}k - \omega_0\right)}}$$

(c) $x[n] =$ 

$\times \left(e^{j\omega_0 n}\right)$

length $= N = 2L+1$, $L = \frac{N-1}{2}$

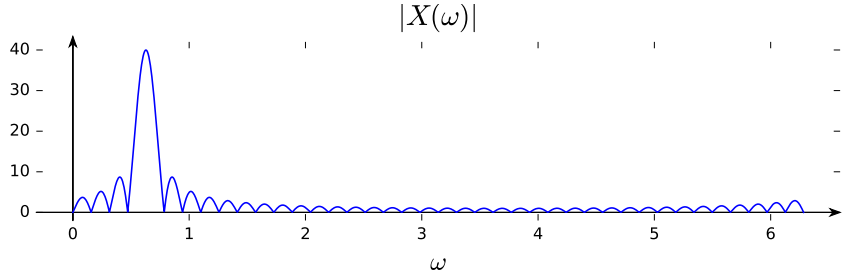
L sample-shift $\longleftrightarrow$ $\times e^{-j\omega L}$

rectangular win $\longleftrightarrow$ $\dfrac{\sin(\pi N f\omega)}{\sin(\pi f\omega)}$

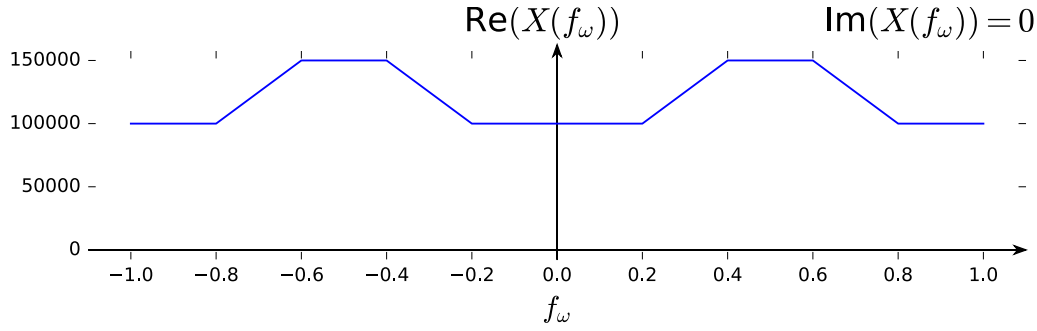
$e^{j\omega_0 n}$ $\longleftrightarrow$ freq delay of ω_0

$$\therefore X(\omega) = e^{-j(\omega-\omega_0)\frac{N-1}{2}} \left(\frac{\sin(N(\omega-\omega_0)/2)}{\sin((\omega-\omega_0)/2)}\right)$$

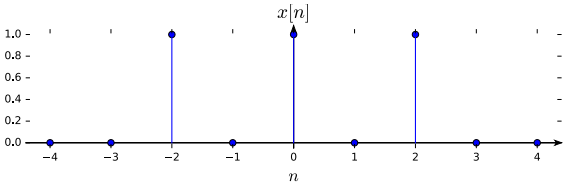
(D)



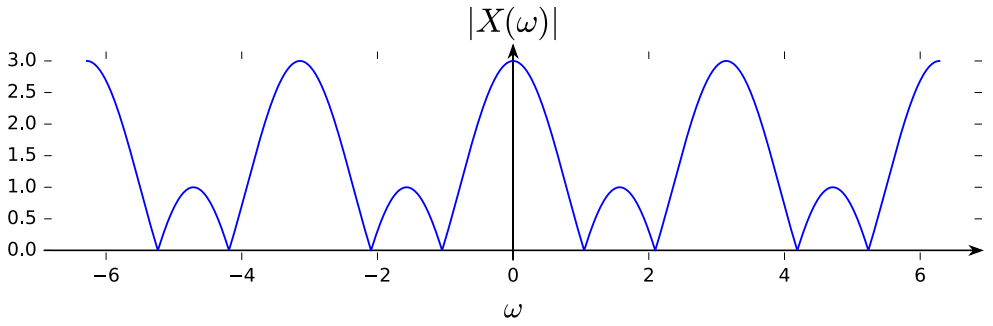
Question 5

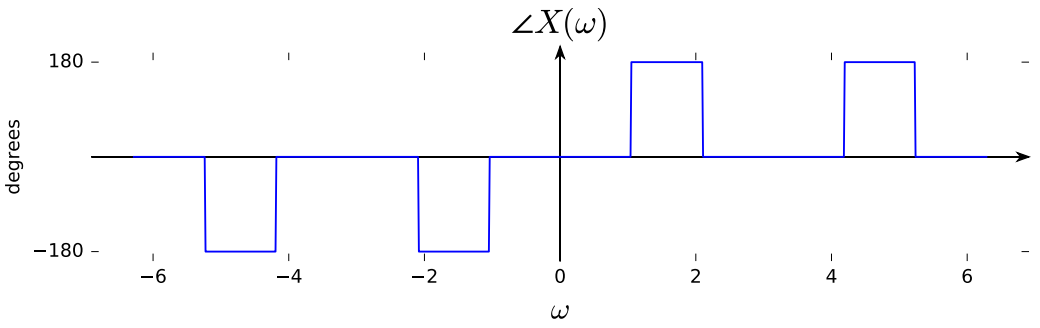


Additional Question 1



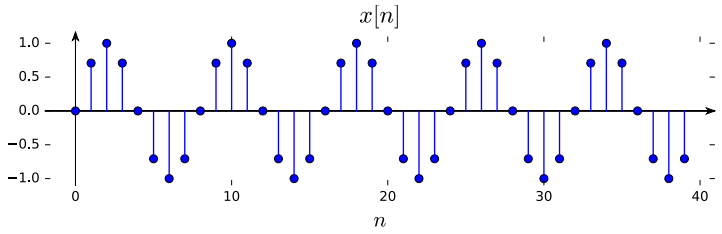
$$\begin{aligned} X(\omega) &= e^{j\omega 2} + e^{-j\omega 0} + e^{-j\omega 2} \\ &= 1 + (e^{j\omega 2} + e^{-j\omega 2}) \\ &= 1 + 2\cos(2\omega)) \end{aligned}$$

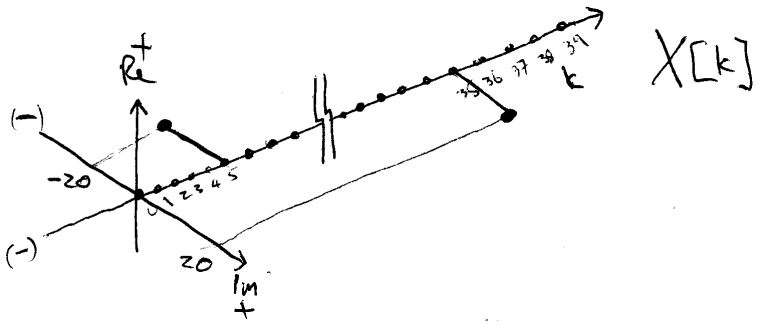




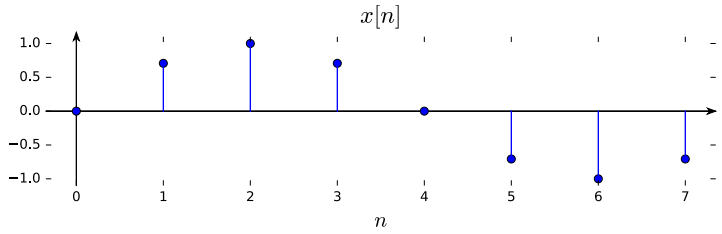
Additional Question 2

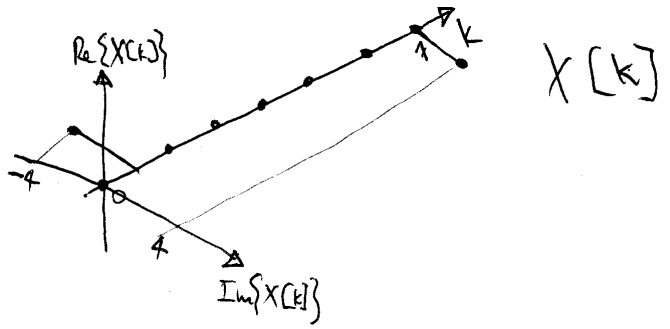
(a)





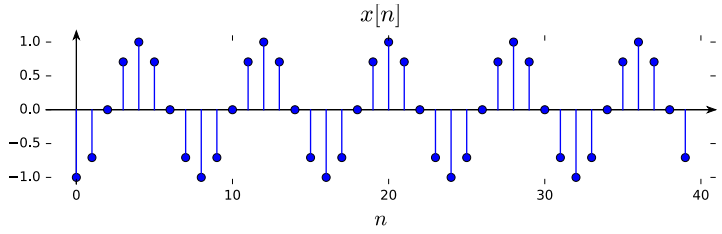
(b)

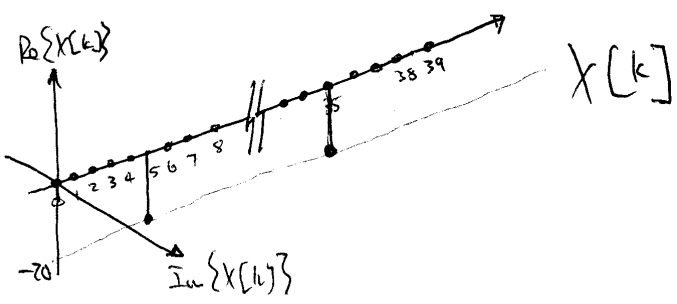




(c) The same as (a)

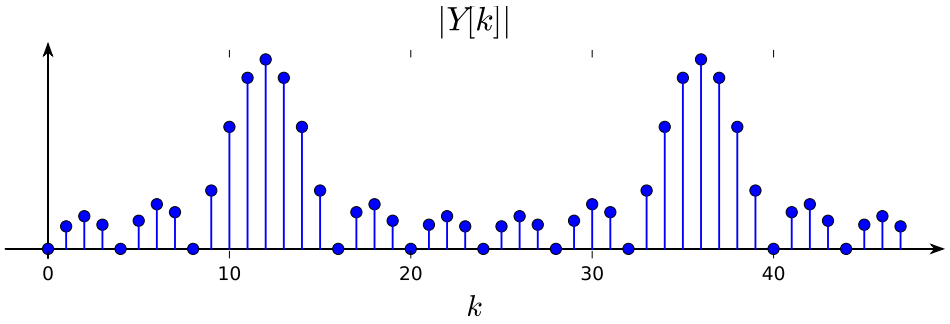
(d)





Additional Question 3

(a)



(b)