and parameter variations. If the performance is not satisfactory, return to Step 1 and repeat.

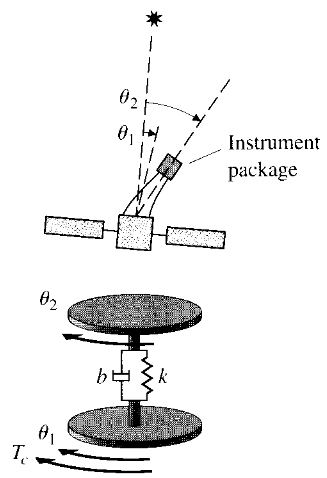
8. Build a prototype and test it. If not satisfied, return to Step 1 and repeat.

9.2 Design of a Satellite's Attitude Control

Our first example, taken from the space program, is suggested by the need to control the pointing direction, or attitude, of a satellite in orbit about the earth. We will go through each step in our design outline and touch on some of the factors that might be considered for the control of such a system.

STEP 1. *Understand the process and its performance specifications.* A satellite is sketched in Fig. 9.2. We imagine that the vehicle has an astronomical survey mission requiring accurate pointing of a scientific sensor package. This package must be maintained in the quietest possible environment, which entails isolating it from the vibrations and electrical noise of the main service body and from its power supplies, thrusters, and communication gear. We model the resulting structure as two masses connected by a flexible boom. Disturbance torques due to solar pressure, micrometeorites, and orbit perturbations are computed to be negligible. The pointing requirement arises when it is necessary to point the unit in another direction, it can be met by dynamics with a transient settling time of 20 sec and an overshoot of no more than 15%. The

FIGURE 9.2
Diagram of a satellite and its two-body model



dynamics of the satellite include parameters that can vary. The control must be satisfactory for any parameter values in a prespecified range to be given when the equations are written.

STEP 2. *Select a sensor*. In order to orient the scientific package, it is necessary to measure the attitude angles of the vehicle. For this purpose we propose to use a **star tracker**, a system based on gathering an image of a specific star and keeping it centered on the focal plane of a telescope. This sensor gives a relatively noisy but very accurate (on the average) reading proportional to θ_2, the angle of deviation of the instrument package from the desired angle. To stabilize the control, we include a rate gyro to give a clean reading of $\dot{\theta}_2$, since a lead network on the star-tracker signal would amplify the noise too much. Furthermore, the rate gyro can stabilize large motions before the star tracker has acquired the target star image.

STEP 3. *Select an actuator*. Major considerations in selecting the actuator are precision, reliability, weight, power requirements, and lifetime. Alternatives for applying torque are cold-gas jets, reaction wheels or gyros, magnetic torquers, and a gravity gradient. The jets have the most power and are the least accurate. Reaction wheels are precise but can only transfer momentum, so jets or magnetic torquers are required to "dump" momentum from time to time. Magnetic torquers provide relatively low levels of torque and are only suitable for some low-altitude satellite missions. A gravity gradient also provides a very small torque that limits the speed of response and places severe restrictions on the shape of the satellite. For purposes of this mission, we select cold-gas jets as being fast and adequately accurate.

STEP 4. *Make a linear model*. For the satellite we assume two masses connected by a spring with torque constant k and viscous-damping constant b as shown in Fig. 9.2. The equations of motion are

$$J_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = T_c, \tag{9.1a}$$

$$J_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0, \tag{9.1b}$$

where T_c is the control torque on the main body. With inertias $J_1 = 1$ and $J_2 = 0.1$, the transfer function is

$$G(s) = \frac{10bs + 10k}{s^2(s^2 + 11bs + 11k)}. \tag{9.2}$$

If we choose

$$\mathbf{x} = [\theta_2 \quad \dot{\theta}_2 \quad \theta_1 \quad \dot{\theta}_1]^T$$

as the state vector, then, using Eq. (9.1) and assuming $T_c \equiv u$, the equations of motion in state-variable form are

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_2} & -\frac{b}{J_2} & \frac{k}{J_2} & \frac{b}{J_2} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_1} & \frac{b}{J_1} & -\frac{k}{J_1} & -\frac{b}{J_1} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_1} \end{bmatrix} u, \tag{9.3a}$$

$$y = [1 \quad 0 \quad 0 \quad 0]\mathbf{x}. \tag{9.3b}$$

Physical analysis of the boom leads us to assume that the parameters k and b vary as a result of temperature fluctuations but are bounded by

$$0.09 \leqslant k \leqslant 0.4 \tag{9.4a}$$

$$0.038\sqrt{\frac{k}{10}} \leqslant b \leqslant 0.2\sqrt{\frac{k}{10}}. \tag{9.4b}$$

As a result, the vehicle's natural resonance frequency ω_n can vary between 1 and 2 rad/sec, and the damping ratio ζ varies between 0.02 and 0.1.

Selecting nominal values for varying parameters

One approach to control design when parameters are subject to variation is to select nominal values for the parameters, contruct the design for this model, and then test the controller performance with other parameter values. In the present case we choose nominal values of $\omega_n = 1$ and $\zeta = 0.02$. The choice is somewhat arbitrary, being based on experience and heuristic analysis. However, note that these are the lowest values in their respective ranges and thus correspond to the plant that is probably the most difficult to control so as to meet the specifications. We assume that a design for this model has a good chance to meet the specifications for other parameter values as well. (Another choice would be to select a model with average values for each parameter). The selected parameter values are $k = 0.091$ and $b = 0.0036$; with $J_1 = 1$ and $J_2 = 0.1$, the nominal equations become

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.91 & -0.036 & 0.91 & 0.036 \\ 0 & 0 & 0 & 1 \\ 0.091 & 0.0036 & -0.091 & -0.0036 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \tag{9.5a}$$

$$y = [1 \quad 0 \quad 0 \quad 0]\mathbf{x}. \tag{9.5b}$$

The corresponding transfer function is then

$$G(s) = \frac{0.036(s + 25)}{s^2(s^2 + 0.04s + 1)}. \tag{9.6}$$

When a trial design is completed, the computer simulation should be run with a range of possible parameter values to ensure that the design has

FIGURE 9.3
Root locus of $KG(s)$

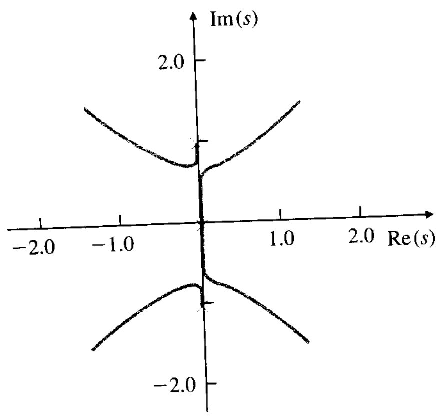
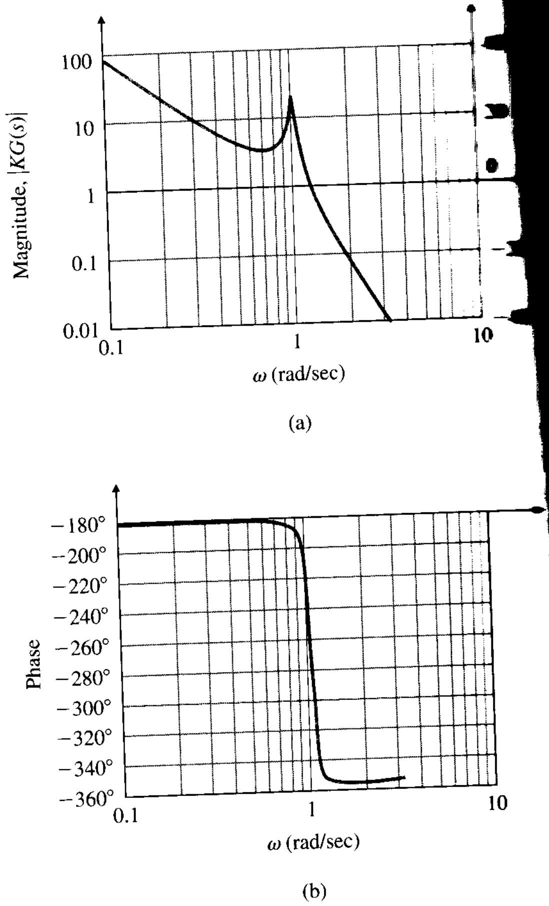


FIGURE 9.4
Open-loop Bode plot of $KG(s)$ for $K = 0.5$



adequate robustness to tolerate these changes. Equations (3.42) tell us that the dynamic performance specifications will be met if the closed-loop poles have a natural frequency of 0.5 rad/sec and a closed-loop damping ratio of 0.5; these coprrespond to an open-loop crossover frequency of $\omega_c \cong 0.5$ rad/sec and a phase margin of about PM = 50°. We will try to meet these design criteria.

STEP 5. *Try a lead-lag or PID controller.* The proportional-gain root locus for the nominal plant is drawn in Fig. 9.3, and the Bode plot is given in Fig. 9.4. We can see from Fig. 9.4 that this may be a difficult design problem because the frequency of the lightly damped resonance is greater than the crossover-frequency design point by only a factor of 2. This situation will require that the compensation can correct for the phase lag of the plant at the