RADIO ASTRONOMY
CORE IDEAS: LECTURE 2 — RADIO ASTRONOMY ESSENTIALS

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1. Brightness distributions

1.1. Basic relations.

- Objects of interest are may be either *point sources*, or have *extended structure* on the sky.
- *Brightness* of sky, B is basic aim of observations.
- Sky brightness or surface brightness measured as infinitesimal power dW per unit area per solid angle $d\Omega$ per unit frequency: $\mathrm{W} \cdot \mathrm{m}^{-2} \cdot \mathrm{Hz}^{-1} \cdot \mathrm{rad}^{-2}$. See Fig. 1.
- Generally, $B(\theta, \phi, \nu)$ — in interferometry, θ and ϕ are usually replaced by direction cosines l and m. (Physics $\nu = f$ engineering)
- B can be integrated over area, solid angle, frequency etc.; if constant over the variable of integration, simple results can be obtained.
- When measured with an antenna with normalised power pattern $P_n(\theta, \phi)$ (see Fig. 2.), output per polarisation is:

$$W = \frac{1}{2} \iint_{\Omega} B(\theta, \phi) P_n(\theta, \phi) d\Omega \tag{1}$$

- Factor 1/2 derives from random polarisation of most RA sources.
- *Beam solid angle* or *beam area* Ω_A defined for $\Omega = 4\pi$ and B unity:

$$\Omega_A = \iint_{4\pi} P_n(\theta, \phi) d\Omega \tag{2}$$

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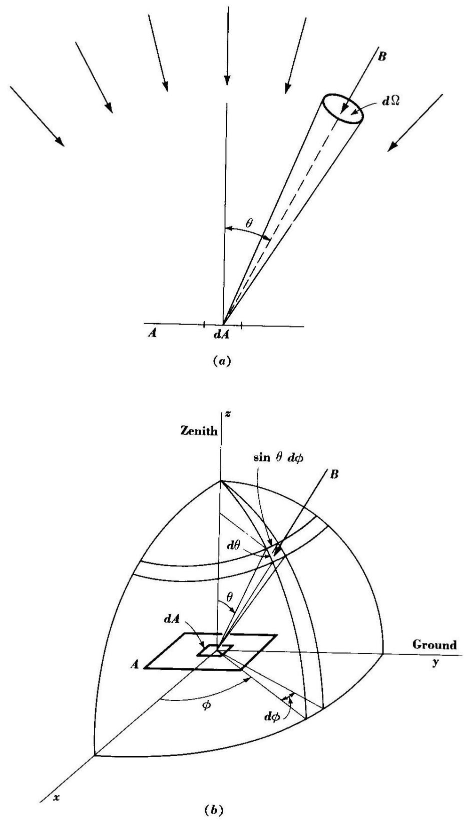

FIGURE 1. Basic geometry for radiation of brightness B, after [1, Fig. 3.1].

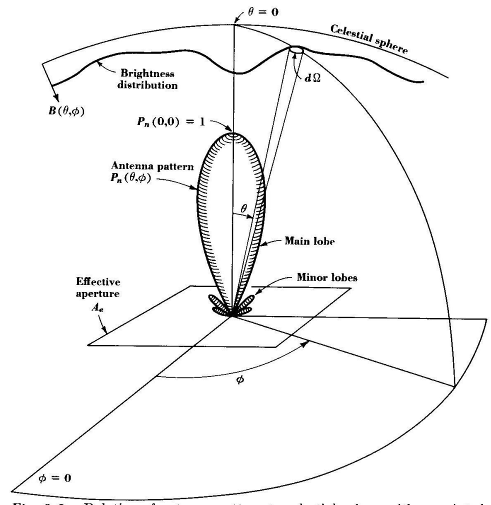

Fig. 3-2. Relation of antenna pattern to celestial sphere with associated coordinates.

FIGURE 2. Relation of antenna patter to celestial sphere with associated coordinates, after [1, Fig. 3.2].

1.2. Discrete sources.

- Sources may: point sources, localised, or extended.
- When resolution of telescope (approx D/λ) is significantly smaller than source extent, source is *resolved*; conversely, for very small sources, they are *unresolved*.
- Angular integral of B gives source *flux density* S:

$$S = \frac{1}{2} \iint B(\theta, \phi) d\Omega \tag{3}$$

- Units are $\mathrm{Wm}^{-2}\mathrm{Hz}^{-1}$ - or in Janskys; 1 Jy = $10^{-26}\mathrm{Wm}^{-2}\mathrm{Hz}^{-1}$
- Observed with antenna:

$$S_o = \frac{1}{2} \iint B(\theta, \phi) P_n(\theta, \phi) d\Omega \tag{4}$$

- For small sources, $P_n(\theta, \phi) \approx 1$, $S_o \approx S$.
- For extended linear sources (see Fig. 3),

$$S_o(\phi_0) = \frac{1}{2} \int B(\phi) P_n(\phi - \phi_0) d\Omega \tag{5}$$

- This is *cross-correlation* of brightness distribution and antenna pattern. Can also be expressed as *convolution* of B and mirror image of P_n.
- Result is that observed flux is a "smeared out" version of B — see Fig. 3.
- The above were far more limiting for single dish systems than for modern interferometric arrays and VLBI (Very Long Baseline Interferometry); interferometers provide very high resolution indeed. However, the basic ideas of convolving the antenna pattern with the source brightness remain core to *imaging*.

1.3. Intensity.

- For an emitting source, appropriate quantity is intensity, I (a.k.a. radiance in the IR).
- Units are the same as B, and it can be shown that

$$I = -B \tag{6}$$

- Also often used in RA.

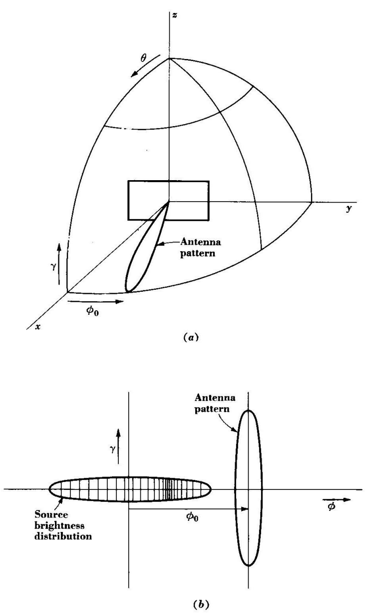

Fig. 3-3. Antenna pattern (*a*) and geometry for long source and fan-beam antenna pattern (*b*).

FIGURE 3. Antenna pattern (a) and geometry for long source and fan-beam antenna pattern (b), after [1, Fig. 3.3].

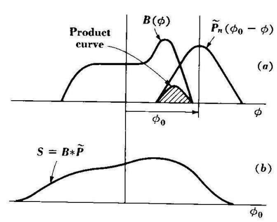

Fig. 3-4. The true brightness distribution B scanned by, or convolved with, the antenna pattern $\tilde{P}$, as in (*a*) yields the observed flux-density distribution S, as in (*b*).

FIGURE 4. The result of convolving the antenna pattern with the true brightness disctribution, after [1, Fig. 3.4].

1.4. **Blackbody radiation and Planck's radiation law.**

- All warm bodies ($T > 0K$) emit radiation.
- Theoretical model is a *blackbody*, which is a perfect absorber and radiator.

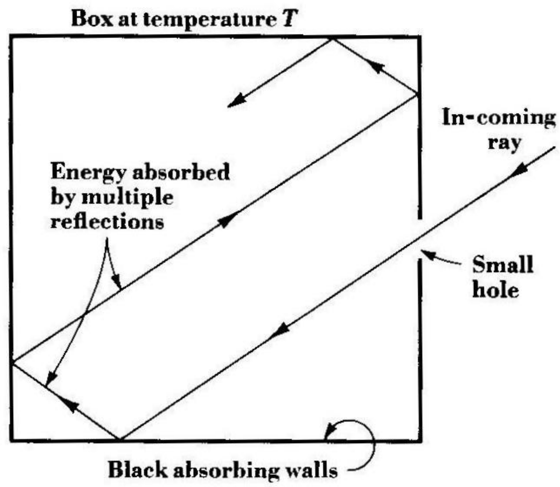

FIGURE 5. A blackbody enclosure, after [1, Fig. 3.11].

- At long wavelengths (low frequencies), emitted $B \propto f^2$.
- UV catastrophe — predicts infinite energy, not the case.
- Planck (1901) resolved this by proposing that a radiation occupies only discrete (quantised) set of possible states.
- *Energy* is quantised, magnitude $h\nu$. Planck's constant $h = 6.63^{-23}\text{J} \cdot \text{s}$.
- From this follows Planck's radiation law:

$$B = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \tag{7}$$

- Wavelength of peak brightness, λ_m:

$$\lambda_m T \approx \frac{hc}{2k} = 0.0048 \text{ m °K} \tag{8}$$

1.5. **Stefan-Boltzman law.**

- Integrating B over all ν gives total brightness B'. Result is:

$$B' = \sigma T^4 \tag{9}$$

with $\sigma = 1.80 \times 10^{-8} W m^{-2} \, {}^oK^{-4}$.

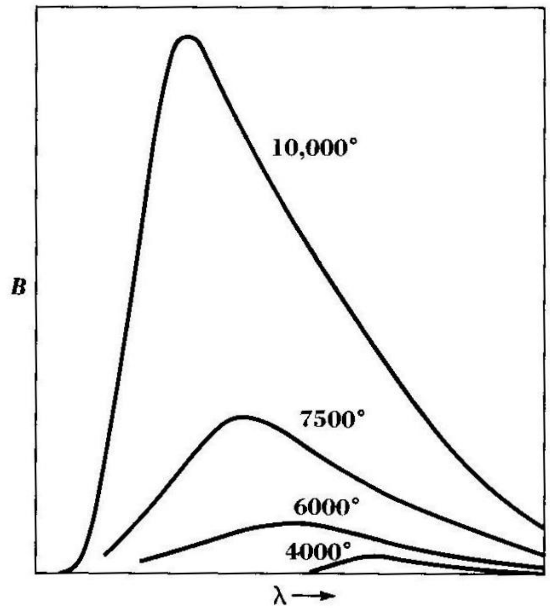

FIGURE 6. Planck radiation law curves for a blackbody radiator at four temperatures, after [1, Fig. 3.12].

1.6. **Rayleigh Jeans law.**

- At RF, $h\nu \ll kT$. Using Taylor series expansion, $e^{h\nu/kT} - 1 \approx 1 + \frac{h\nu}{kT} - 1 = \frac{h\nu}{kT}$. Thus

$$B \approx \frac{2kT}{\lambda^2} \tag{10}$$

the low frequency approximation of classical physics, pre-Planck.
- There is a similar high-frequency approximation (the Wien radiation law):

$$B \approx \frac{2h\nu^3}{c^2} e^{-h\nu/kT} \tag{11}$$

- These approximate different regions of the full Planck law. See Fig. 7.

1.7. **Temperature and noise.**

- Thermal noise in the receiving system is the ultimate limit on sensitivity.
- Thermal (Johnson) noise characterised by

$$w \approx kT \tag{12}$$

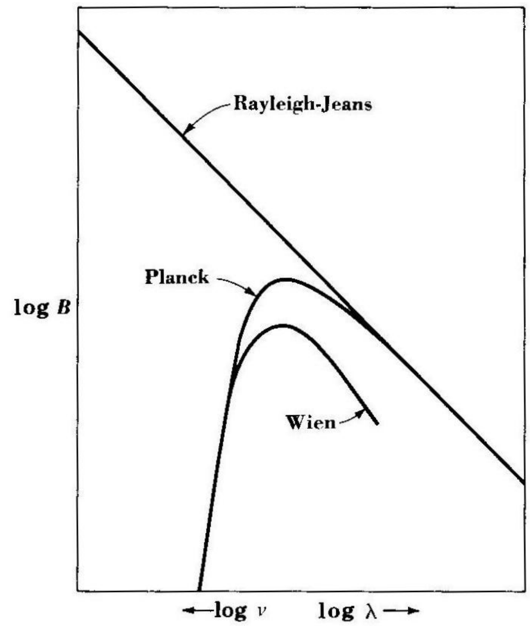

FIGURE 7. The Planck radiation law, compared with the Rayleigh-Jeans and Wien approximations, after [1, Fig. 3.17].

with w the power per unit BW, W/Hz.

- Note that this results relies on the Rayleigh-Jeans approximation.
- An antenna looking at a region in the sky of average temperature T receives the same power as if placed inside a blackbody at T, or connected to a resistor at T.
- With T_A the measured antenna temperature (the sky temperature if the antenna is lossless), observed flux density is

$$S_o = \frac{2kT_A}{A_e} \tag{13}$$

with A_e the effective receiving area of the antenna.

- NB! If the source is actually the result of thermal radiation, then T_A may the actual blackbody temperature.
- Many RA sources are non-thermal, resulting in much "hotter" equivalent temperatures.

1.8. **Minimum detectable temperature and flux density.**

- T_{sys}, the noise in the receiver chain, limits the minimum detectable antenna temperature. Usually dominated by first element in receiver chain.

- Sensitivity defined as r.m.s. noise temperature of system:

$$\Delta T_{\rm min} = \frac{K_s T_{\rm sys}}{\sqrt{\Delta\nu\, t\, n}} \tag{14}$$

with K_s a receiver-dependent sensitivity constant (close to unity); $\Delta\nu$ is bandwidth; t integration time; n number of records averaged (often $t\,n$ replaced by only t since processes of integrated and averaging are very similar).

- Using Rayleigh-Jeans, minimum detectable brightness follows easily:

$$\Delta B_{\rm min} = \frac{2k}{\lambda^2}\frac{K_s T_{\rm sys}}{\sqrt{\Delta\nu\, t\, n}} \tag{15}$$

and from $S_o = \frac{2kT_A}{A_e}$, we find *minimum detectable flux density*:

$$\Delta S_{\rm min} = \frac{2k}{A_e}\frac{K_s T_{\rm sys}}{\sqrt{\Delta\nu\, t\, n}} \tag{16}$$

usually given in Jy.

- Parameter $A_e/T_{\rm sys}$ is fundamental in radio astronomy (the square of this enters in the survey speed equation) — for MeerKAT it is close to $300{\rm m}^2/{\rm K}$.
- System equivalent flux density, S_E, or $S_{\rm SEFD}$, is another:

$$S_E = \frac{2kT_{\rm sys}}{A_e} \tag{17}$$

also usually given in Jy. (Interpretation: S_E is flux density of source in main beam producing twice system noise power).

- Recently, (point source) survey speed figure of merit has been introduced for new survey instruments (e.g. SKA)

$$\begin{aligned} {\rm PFoM} &\equiv \frac{2n_p\,\Delta\nu\,\Omega_{\rm FoV}}{{S_E}^2} \\ &= \frac{n_p\,\Delta\nu\,\Omega_{\rm FoV}}{2k^2}\left(\frac{A_e}{T_{\rm sys}}\right)^2 \end{aligned} \tag{18}$$

$\Omega_{\rm FoV}$ is instantaneous field of view of instrument; n_p is number of polarisations (usually 2). Note that $A_e/T_{\rm sys}$ enters as *square*.

References

[1] J. D. Kraus, *Radio Astronomy.* New York: McGraw-Hill, 1966.