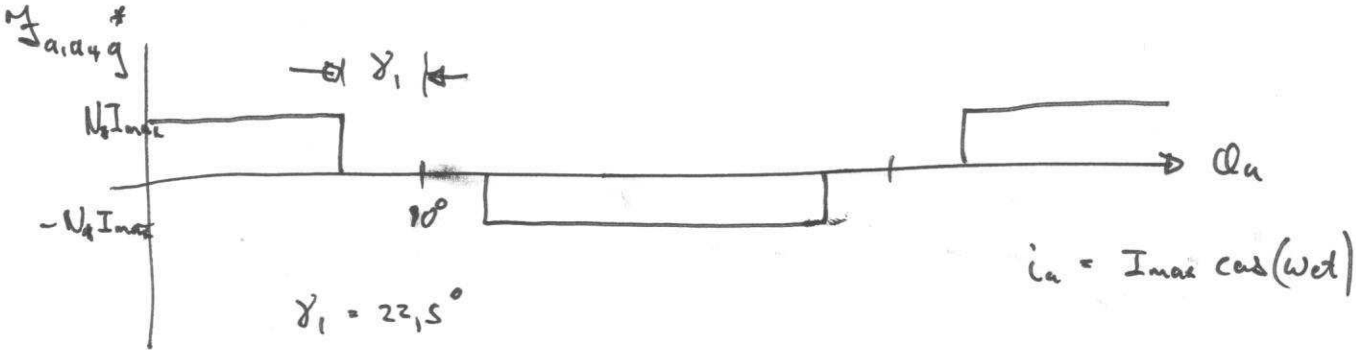
Tutorial III ES344

Question 1:

a) See Notes 31/143

b) Use superposition and divide the MMF produced by the coils in slots $-a_1|a_1$, $-a_2|a_2$, $-a_3|a_3$ & $-a_4|a_4$ into $-a_1|a_1$ & $-a_4|a_4$ and $-a_2|a_2$ & $-a_3|a_3$

See Notes 33/143 * & 34/143 **



This waveform is ¼ wave symmetrical (see also Notes 21/143)

$$\Rightarrow \quad a_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{F}_{a_1a_4}(\theta_a)\cos(n\theta_a)\,d\theta_a \quad \text{For } n = 1,3,5\ldots$$

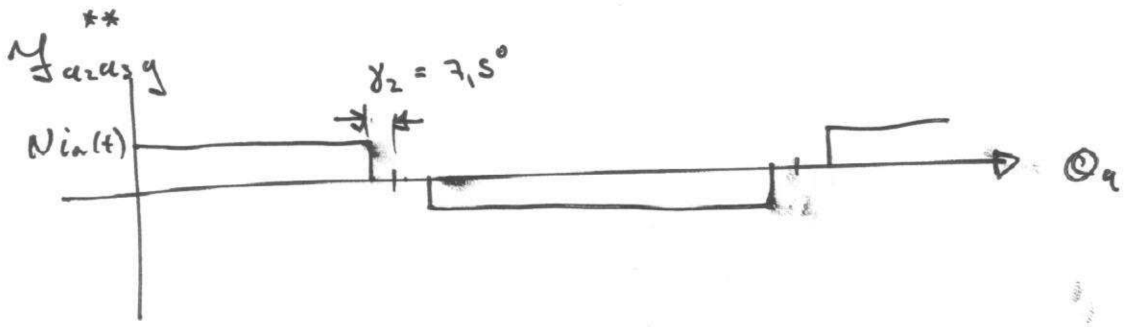
$$= \frac{4}{\pi}\int_0^{\frac{\pi}{2}-\gamma_1} N_s I_{max}\cos(\omega_e t)\cos(n\theta_a)\,d\theta_a$$

$$= \frac{4}{\pi} N_s I_{max}\cos(\omega_e t)\left[\frac{\sin(n\theta_a)}{n}\right]_0^{\frac{\pi}{2}-\gamma_1}$$

$$= \frac{4}{\pi} N I_{max}\cos(\omega_e t)\,\frac{\sin\left(n\frac{\pi}{2} - n\gamma_1\right)}{n}$$

$$= \frac{4}{\pi} N I_{max}\cos(\omega_e t)\,\frac{\sin\left(n\frac{\pi}{2}\right)\cos(n\gamma_1) - \underbrace{\cancel{\cos\left(n\frac{\pi}{2}\right)}}_{=0 \text{ for } n=1,3,5\ldots}\sin(n\gamma_1)}{n}$$

$$\Rightarrow \quad a_n' = \frac{4}{\pi}\, N\, i_a(t)\, \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \cos(n\gamma_1)$$



ditto

$$a_n'' = \frac{4}{\pi}\, N\, i_a(t)\, \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \cos(n\gamma_2)$$

$$\overset{a_n}{\cancel{\mathcal{F}_{ag}}} = \overset{a_n'}{\cancel{\mathcal{F}_{a1a4g}}} + \overset{a_n''}{\cancel{\mathcal{F}_{a2a3g}}}$$

$$= \frac{4}{\pi}\, N\, i_a(t)\, \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \Big[\cos(n\gamma_1) + \cos(n\gamma_2) \Big] \quad -\ 1$$

But $N_{ph} = 4 \quad \Rightarrow \quad 1 \times \frac{4}{4}$

$$\Rightarrow \quad \overset{a_n}{\cancel{\mathcal{F}_{ag}}} = \frac{4}{\pi}\left(\frac{N_{ph}\, i_a(t)}{2}\right) \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \left[\frac{\cos(n\gamma_1) + \cos(n\gamma_2)}{2} \right]$$

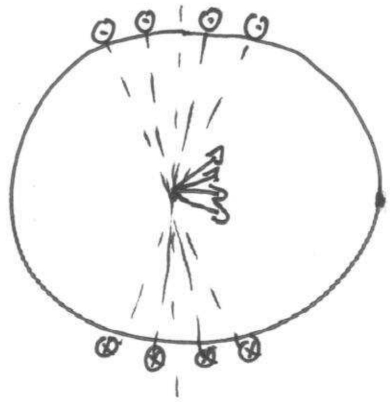
$$\Rightarrow \quad \mathcal{F}_{ag} = \sum_{n=1,3,5}^{\infty} a_n \cos(n\theta_a)$$

$$= \frac{4}{\pi}\left(\frac{N_{ph}\, i_a(t)}{2}\right) \sum_{n=1,3,5\ldots}^{\infty} \frac{\sin\left(n\frac{\pi}{2}\right)}{n} k_{wn} \cos(n\theta_a)$$

with $$k_{wn} = \frac{\cos(n\gamma_1) + \cos(n\gamma_2)}{2}$$

c) See k_{wn} above.

d)



see Notes 39/143 & 40/143

For calculations see Notes 41/143 → 42/143

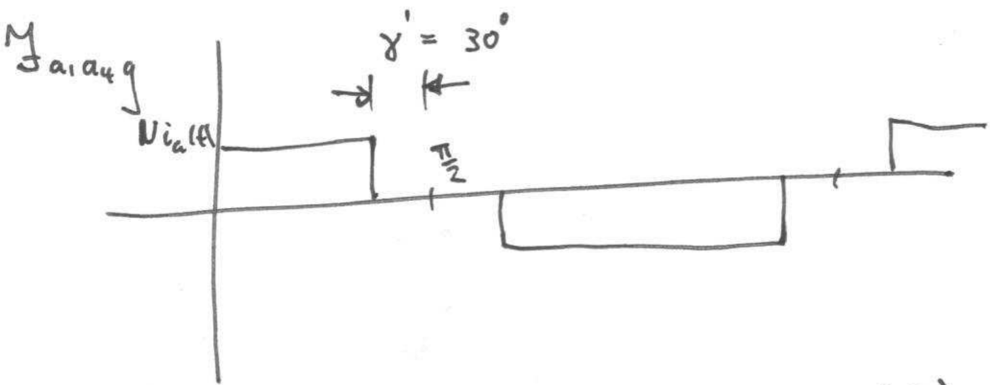
$$K_{w1} = \frac{\cos(\gamma_1) + \cos(\gamma_2)}{2}$$

Question 2:

a) See Notes 47/143

b) Use superposition and divide the MMF produced by the coils $-a_1/a_1 \rightarrow -a_4/a_4$ into:

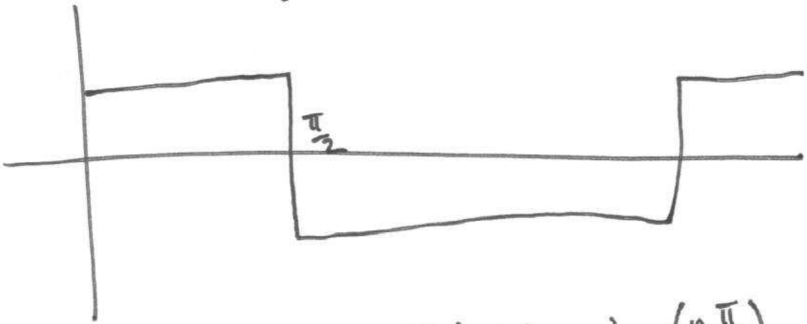
$-a_1/a_1$ & $-a_4/a_4$ and $-a_2/a_2$ & $-a_3/a_3$



From Q1b) $a_n' = \frac{4}{\pi} N i_a(t) \frac{\sin(n\frac{\pi}{2})}{n} \cos(n\gamma')$

$\gamma'' = 0$

$\mathcal{F}_{a_2 a_3 q}$



$\cos(0°) = 1$

$$\Rightarrow \quad \cancel{\mathcal{F}_a} \quad a_n'' = \frac{4}{\pi} N i_a(t) \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \cdot 1$$

$$\Rightarrow \quad a_n = a_n' + a_n''$$

$$= \frac{4}{\pi} N i_a(t) \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \left[\cos(n\gamma') + 1 \right] \quad (2)$$

But $N_{ph} = 4N \Rightarrow (2) \times \frac{4}{4}$

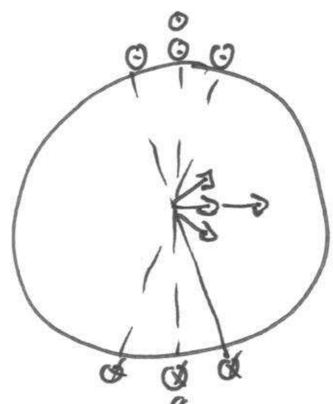
$$\therefore \quad a_n = \frac{4}{\pi} \left(\frac{N_{ph}\, i_a(t)}{2} \right) \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \left[\frac{\cos(n\gamma') + 1}{2} \right]$$

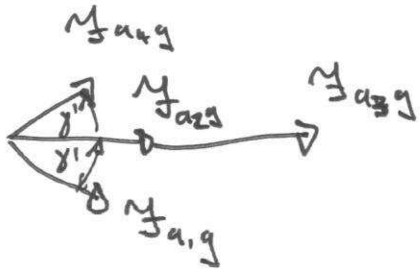
$$\Rightarrow \quad \mathcal{F}_{ag} = \frac{4}{\pi} \left(\frac{N_{ph}\, i_a(t)}{2} \right) \sum_{n=1,3,5\ldots}^{\infty} \frac{\sin\left(n\frac{\pi}{2}\right)}{n} k_{wn} \cos(n\theta_a)$$

with $k_{wn} = \dfrac{\cos(n\gamma') + 1}{2}$

c) see k_{wn} above

d)





$$\therefore \quad \vec{\mathcal{F}}_{ag_1} = \mathcal{F}_{a2g}\angle 0^\circ + \mathcal{F}_{a3g}\angle 0^\circ + \mathcal{F}_{a1g}\angle -\gamma' + \mathcal{F}_{a4g}\angle \gamma'$$

$$= \frac{4}{\pi}\left(\frac{N i_a(t)}{2}\right)\left[2 + 2\cos(\gamma')\right]\angle 0^\circ$$

$$\therefore \quad \mathcal{F}_{ag_1} = \frac{4}{\pi}\left(\frac{N i_a(t)}{2}\right)\left[2 + 2\cos(\gamma')\right] \times \frac{4}{4}$$

$$\because \quad N_{ph} = 4N$$

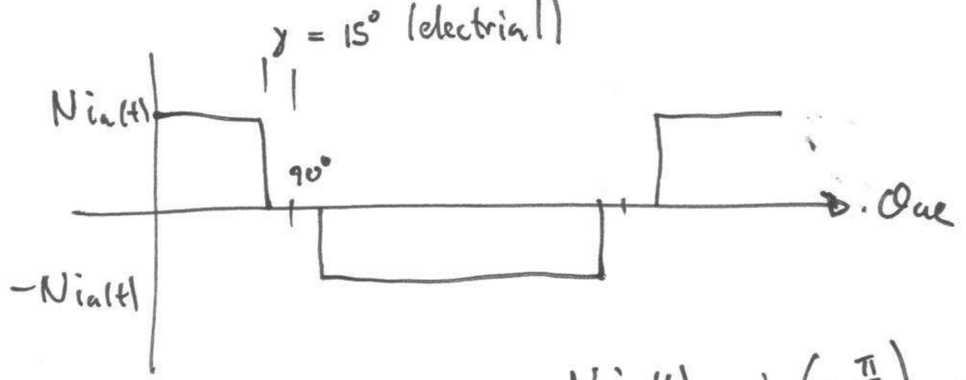
$$\Rightarrow \quad \mathcal{F}_{ag_1} = \frac{4}{\pi}\left(\frac{N_{ph}(i_a(t))}{2}\right)\left[\frac{1 + \cos(\gamma')}{2}\right]$$

$$\Rightarrow \quad K_{w_1} = \frac{\cos(\gamma') + 1}{2} \qquad [n = 1]$$

Question 3:

a) See Notes 62/143

b) Only draw MMF for one 360° (electrical) cycle



$$\Rightarrow \quad a_n = \frac{4}{\pi} N i_a(t) \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \cos(n\gamma)$$

$$\Rightarrow \quad \mathcal{F}_{ag} = \frac{4}{\pi} N i_a(t) \sum_{n=1,3,5.}^{\infty} \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \cos(n\gamma) \cos(n\theta_{ae})$$

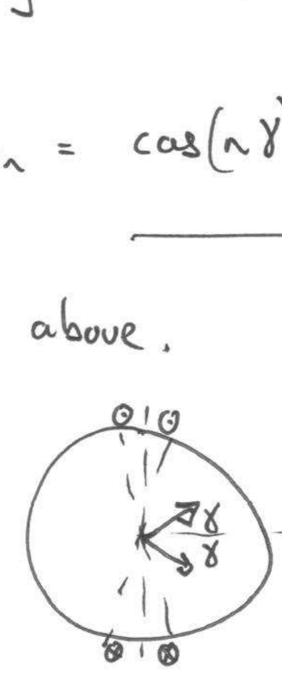
But $N_{ph} = 4N$ & $p = 2$

$$\Rightarrow \quad \mathcal{F}_{ag} = \frac{4}{\pi} \left(\frac{N_{ph}\, i_a(t)}{2p}\right) \sum_{n=1,3,5..}^{\infty} \frac{\sin\left(n\frac{\pi}{2}\right)}{n} k_{wn} \cos(n\theta_{ae})$$

with $k_{wn} = \cos(n\gamma)$ & $\gamma = 15°$ (elec.)

c) See k_{wn} above.

d)



θ_{ae} (Virtual elec [°] equiv.)

$$\vec{\mathcal{F}}_{ag_1} = \frac{4}{\pi}\left(\frac{N}{2}\right) i_a(t) \angle \gamma + \frac{4}{\pi}\left(\frac{N}{2}\right) i_a(t) \angle -\gamma$$

$$= \frac{4}{\pi}\left(\frac{N i_a(t)}{2}\right) 2 . \cos(\gamma) \angle 0°$$

$N_{ph} = 4N$

$$\therefore \quad \mathcal{F}_{ag_1} = \frac{4}{\pi}\left(\frac{N_{ph}\, i_a(t)}{2p}\right) \cos(\gamma)$$

with $\cos\gamma = k_{w_1}$

& $p = 2$

Question 4:

a) See Notes 67/143

b) Similar to Q 3b) but in electrical [°]

$$a_n = a_n' + a_n''$$

$$= \frac{4}{\pi} N i_a(t) \frac{\sin\left(n\frac{\pi}{2}\right)}{n} \left[\cos(n\gamma) + 1\right]$$

But $N_{ph} = 8N$ & $p = 2$

$$\therefore \quad a_n = \frac{4}{\pi}\left(\frac{N_{ph}\, i_a(t)}{2p}\right) \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \left[\frac{\cos(n\gamma) + 1}{2}\right]$$

$$\Rightarrow \quad \mathcal{F}_{ag} = \frac{4}{\pi}\left(\frac{N_{ph}\, i_a(t)}{2p}\right) \sum_{n=1,3,5}^{\infty} \frac{\sin\left(n\frac{\pi}{2}\right)}{n} k_{wn} \cos(n\theta_{ae})$$

With $k_{wn} = \dfrac{\cos n\gamma + 1}{2}$ & $\gamma = 30°$ (electr.)

c) See hwn of (b)

d) Same as (d) of Q2 we just work in a virtual elect. [°] equivalent plane.