



FAKULTEIT INGENIEURSWESE  
FACULTY OF ENGINEERING



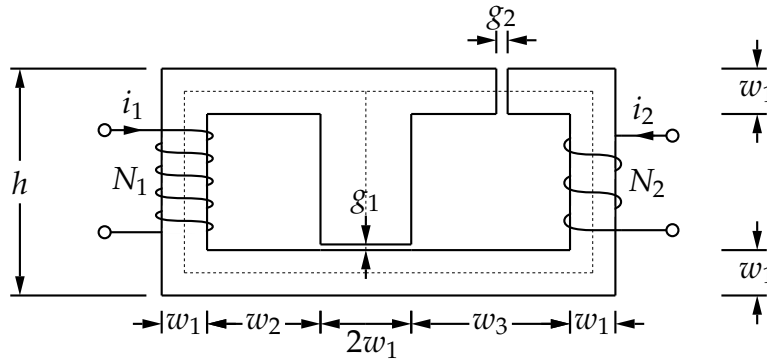
<b>Energiestelsels 344</b>		<b>Energy Systems 344</b>	
<b>Asseseringstoets I</b> <i>Assesment Test I</i>	<b>3 September 2015</b> <i>3 September 2015</i>	<b>Tydsduur:</b> <i>Duration:</i> <b>2h</b>	<b>Totaal:</b> <i>Total:</i> <b>68</b>

<b>Eksaminator</b> <i>Examiner</i>	<b>Interne Moderator</b> <i>Internal Moderator</i>
Dr. P.J. Randewijk	Prof. M.J. Kamper

<b>Instruksies</b>	<b>Instructions</b>
<ol style="list-style-type: none"><li>1. Alle berekening stappe moet duidelik getoon word.</li><li>2. Maak deurgaans van eenvoudige sketse of ekwivalente stroombaandiagramme gebruik om u antwoorde te illustreer.</li><li>3. Die finale antwoorde moet in inkskryf word.</li><li>4. Punte sal afgetrek word indien antwoorde se eenhede nie getoon word nie, b.v. 123,45 V, 123,45 mA, 123,456 kW, ens.</li><li>5. Werk met ten minste vier (4) beduidende syfers, b.v. 123,4 V, 12,34 V, 1,234 V, ens.</li><li>6. Sakrekenaars soos voorgeskryf vir die eerste twee jaar van B.Ing. mag gebruik word.</li><li>7. As daar, na u mening, enige parameters ontbreek of verkeerd is, maak redelike aannames, verklaar duidelik waarom u aannames gebruik en gaan voort.</li><li>8. Indien daar 'n verskil tussen die "Afrikaanse" en "Engelse" gedeeltes is, stel duidelik watter taal se inligting u gebruik vir u antwoord.</li><li>9. Geen selfone word toegelaat nie.</li></ol>	<ol style="list-style-type: none"><li>1. All calculation steps must be shown clearly.</li><li>2. Continuously make use of simple sketches or equivalent circuit diagrams to illustrate all answers.</li><li>3. The final answers must be written in ink.</li><li>4. Marks will be deducted if no units for answers are shown, e.g. 123,45 V, 123,45 mA, 123,45 kW, etc.</li><li>5. Work with at least four (4) significant numbers, e.g. 123,4 V, 12,34 V, 1,234 V, etc.</li><li>6. Pocket calculators as prescribed for the first two years of B.Eng. may be used.</li><li>7. If, in your opinion, there are any missing or incorrect parameters, make reasonable assumptions, stating them clearly and continue.</li><li>8. If there is a difference between the "Afrikaans" and "English" section, state clearly which language's information you are using for your answer.</li><li>9. No cellphones are allowed.</li></ol>

(a) Vir die onderstaande magnetiese baan met diepte,  $D$ , en relatiewe permeabiliteit,  $\mu_r \rightarrow \infty$ ,

(a) For the magnetic circuit shown below with depth,  $D$ , and relative permeability,  $\mu_r \rightarrow \infty$ ,



- i. teken die ekwivalente magnetiese stroombaan diagram in terme van  $\mathcal{F}_1=N_1i_1$ ,  $\mathcal{F}_2=N_2i_2$  en die minimum aantal reluktansies. [2]
- ii. skryf die vergelyking neer vir elke reluktansie in die ekwivalente stroombaan in terme van die magnetiese baan se dimensies. [2]
- iii. kry 'n vergelyking vir elke induktansie in die induktansie matriks,  $L_{11}$ ,  $L_{12}$ ,  $L_{21}$  en  $L_{22}$  in terme van die bogenoemde reluktansies. [8]

- i. draw the equivalent magnetic circuit diagram, in terms of  $\mathcal{F}_1=N_1i_1$ ,  $\mathcal{F}_2=N_2i_2$  and the minimum number of reluctances. [2]
- ii. write down the equations for each reluctance in the equivalent circuit in terms of the magnetic circuit's dimensions. [2]
- iii. find an equation for each inductance in the inductance matrix,  $L_{11}$ ,  $L_{12}$ ,  $L_{21}$  and  $L_{22}$  in terms of the above mentioned reluctances. [8]

**Wenk:** Gebruik superposisie.

**Tip:** Use superposition.

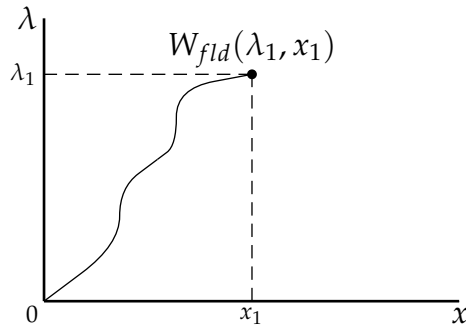
(a) Deur van die energie metode gebruik te maak, kan dit verandering in die gestoorde energie van 'n konserwatiewe stelsel as volg geskryf word,

(a) By making use of the energy method, the change in the stored energy of a conservative system can be written as follows,

$$dW_{fld} = id\lambda - f_{fld}dx \quad \text{met / with} \quad \lambda = L(x)i.$$

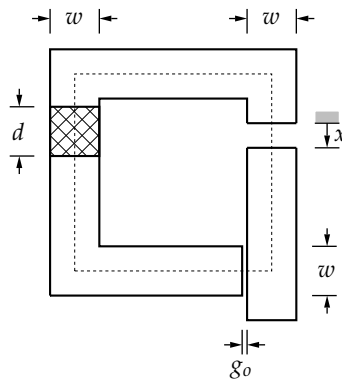
Verduidelik (met woorde en wiskunde) hoe die gestoorde energie in die stelsel,  $W_{fld}(\lambda, x)$ , maklik bereken kan word as 'n funksie van die toestand veranderlikes  $\lambda$  en  $x$  deur die integrasie pad op 'n "slim manier te kies", i.p.v. die werklike pad wat gevolg is, soos bo aan die volgende bladsy getoon.

Explain (using words and maths) how the stored energy in the system,  $W_{fld}(\lambda, x)$ , can easily be calculated as a function of the state variables  $\lambda$  and  $x$  by choosing the integration path in an "intelligent way", instead of the actual path that was followed, as shown on top of the next page.



(b) Met die relatiewe permeabiliteit van die kern,  $\mu_r \rightarrow \infty$ , die terugspring permeabiliteit van die NdFeB permanente magneet,  $\mu_R = -\frac{B_m}{H_c}$ , die diepte van die kern,  $D$ , en  $g_o$  'n vaste lugspleet

(b) With the permeability of the core,  $\mu_r \rightarrow \infty$ , the recoil permeability of the NdFeB permanent magnet,  $\mu_R = -\frac{B_m}{H_c}$ , the depth of the core,  $D$ , and  $g_o$  a fixed air-gap,



- i. teken die ekwivalente magnetiese stroombaan diagram. [2]
- ii. lei 'n vergelyking af vir die krag wat deur die NdFeB magneet op die bewegende kern gedeelte uitgeoefen word as 'n funksie van  $x$ . [6]

- i. draw the equivalent magnetic circuit diagram. [2]
- ii. find an equation for the force exerted by the NdFeB permanent magnet on the moving part of the core as a function of  $x$ . [6]

**Wenk:** Maak van 'n ekwivalente MMK,  $(Ni)_{equiv}$ , gebruik en ignoreer die fraaiings effek.

**Tip:** Make use of an equivalent MMF,  $(Ni)_{equiv}$  and ignore the fringing effect.

**Vraag 3: Roterende MMK**

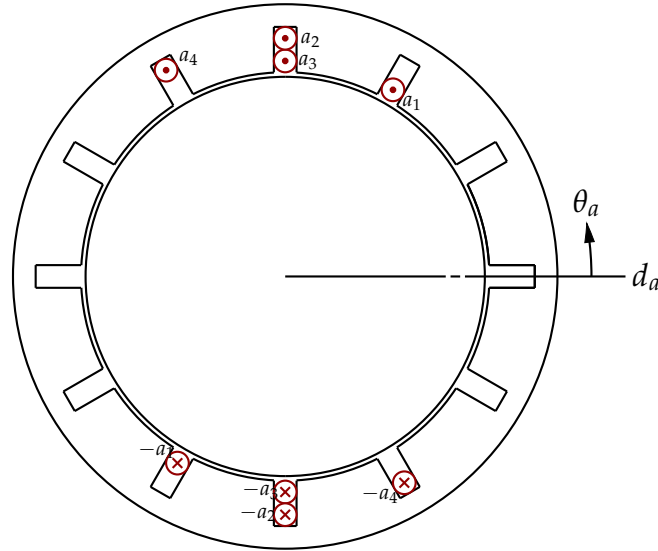
[24]

**Question 3: Rotating MMF**

[24]

Hierdie vraag het betrekking op die stator van 'n dubbellaag sinchroon masjien waarvan slegs die spoel van fase  $a$  op die volgende bladsy getoon word. Die hoeveelheid serie windings per fase,  $N_{ph} = 4N$ , met  $N$  die hoeveelheid windings in elke spoel.

This question has to do with the stator of a double layer synchronous machine with only the coils of phase  $a$  shown on top of the next page. The number of serie turns per phase,  $N_{ph} = 4N$ , with  $N$  the number of turns in each coil.



- (a) Teken die lineêre MMK distribusie in die lugspleet van die masjien vir fase  $a$  vir enige  $i_a(t)$ , vanaf  $0^\circ$  tot  $360^\circ$  (*mechanies*), indien ons aanvaar dat die relatiewe permeabiliteit van die stator en die rotor,  $\mu_r \rightarrow \infty$ . [4]
- (b) Bereken die Fourier reeksuitbreiding van die MMK distribusie in die lugspleet van die masjien vir fase  $a$ ,  $\mathcal{F}_{ag}$ , uit eerste beginsels in terme van die totale aantal serie windings per fase,  $N_{ph}$  en die fase stroom  $i_a(t)$ . [8]

**Wenk:** Maak van superposisie gebruik.

- (a) Draw the linear MMF distribution in the air-gap of the machine for phase  $a$  for any  $i_a(t)$ , from  $0^\circ$  to  $360^\circ$  (*mechanical*), if we assume that the relative permeability of the stator and the rotor,  $\mu_r \rightarrow \infty$ . [4]
- (b) Calculated the Fourier series expansion of the MMF distribution in the air-gap of the machine for phase  $a$ ,  $\mathcal{F}_{ag}$ , from first principles in terms of number of series turns per phase,  $N_{ph}$  and the phase current,  $i_a(t)$ . [8]

**Tip:** Make use of superposition.

- (c) Vanaf u antwoord in (b) hierbo, skryf die algemene vergelyking vir die wikkelingsfaktor,  $k_{w_n}$ , neer vir hierdie wikkelingskonfigurasie. [2]
- (d) Maak van MMK ruimte-vektore gebruik om die fundamentele wikkelingsfaktor,  $k_{w_1}$ , te bereken en gebruik dit om u antwoord in (c) hierbo te bevestig. [4]
- (e) Vanaf die onderste gegewens, lei 'n vergelyking af vir die fundamentele komponent van die resulterende MMK,  $\mathcal{F}_{res,1}$ , met die magnetiese as van fase  $b$  by  $120^\circ$  (*mechanies*) en die magnetiese as van fase  $c$  by  $-120^\circ$  (*mechanies*): [6]

- (c) From your answer in (b) above, write down the general equation for the winding factor,  $k_{w_n}$ , for this winding configuration. [2]
- (d) Make use of MMF space-vectors to calculate the fundamental winding factor,  $k_{w_1}$  and use it to check your answer in (c) above. [4]
- (e) From the information given below, deduce an equation for the fundamental component of the resultant MMF,  $\mathcal{F}_{res,1}$ , with the magnetic axis of phase  $b$  at  $120^\circ$  (*mechanical*) and the magnetic axis of phase  $c$  at  $-120^\circ$  (*mechanical*): [6]

$$\begin{aligned}
 \mathcal{F}_{ag_1} &= \frac{4}{\pi} \frac{k_{w_1} N_{ph}}{2} i_a(t) \cos(\theta_a) \\
 \mathcal{F}_{bg_1} &= \frac{4}{\pi} \frac{k_{w_1} N_{ph}}{2} i_b(t) \cos(\theta_b) \\
 \mathcal{F}_{cg_1} &= \frac{4}{\pi} \frac{k_{w_1} N_{ph}}{2} i_c(t) \cos(\theta_c) \\
 i_a(t) &= I_{max} \cos(\omega_e t) \text{ A} \\
 i_b(t) &= I_{max} \cos(\omega_e t + 120^\circ) \text{ A} \\
 i_c(t) &= I_{max} \cos(\omega_e t - 120^\circ) \text{ A}
 \end{aligned}$$

- (a) Verduidelik aan die hand van 'n eenvoudige skets, wat ankerreaksie is en hoe dit die kommutasie beïnvloed. [3] (a) Explain with the aid of a few simple sketches, what armature reaction is and how it affects commutation. [3]
- (b) Verduidelik, deur onder andere van die  $B-H$  kromme gebruik te maak, hoe dit moontlik is dat 'n newesluitings GS-generator 'n spanning kan opwek. [2] (b) Explain, making use of amongst other the  $B-H$  curve, how it is possible for a shunt connected DC generator to generate a voltage. [2]
- (c) Die veilige  $T_m-\omega_m$  bedryfsarea van 'n afsonderlik gemagnetiseerde GS motor kan opgedeel word in 'n konstante draaimoment gebied en 'n konstante drywingsgebied. (c) The safe  $T_m-\omega_m$  operating area of a separately excited DC motor can be divided into a constant torque region and the constant power region.
- i. Teken die  $T_m-\omega_m$  bedryfsarea van die konstante draaimomentgebied en toon aan en verduidelik die waardes (simbolies) wat die bedryfsgrens definieer. [2] i. Sketch the  $T_m-\omega_m$  operating area of the constant torque region and indicate and explain the values (symbolically) that define the operating boundary. [2]
- ii. Teken die  $T_m-\omega_m$  bedryfsarea van die konstante drywingsgebied op skets (i) hierbo, en toon aan en verduidelik die waardes (simbolies) wat die bedryfsgrens in hierdie gebied definieer. [2] ii. Sketch the  $T_m-\omega_m$  operating area of the constant power region on sketch (i) above, and indicate and explain the values (symbolically) that define the operating boundary. [2]
- iii. met die veldklemspanning konstant, lei 'n vergelyking af vir die ontwikkelde draaimoment as 'n funksie van spoed ( $\omega_m$ ) vir die afsonderlik gemagnetiseerde GS-motor en stip die tipiese ken-lyne van die ontwikkelde draaimoment vir (sê) drie verskillende **ankerklenspanningswaardes** op die veilige bedryfsarea van (i) en/of (ii) hierbo. [4] iii. with the field terminal voltage constant, derive an equation for the developed torque as a function of speed ( $\omega_m$ ) of the separately excited DC-motor and plot the typical characteristic curves of the developed torque for (say) three different **armature terminal voltage values** on the safe operating area of (i) and/or (ii) above. [4]
- (d) Die veilige  $P_m-\omega_m$  bedryfsarea van 'n afsonderlik gemagnetiseerde GS motor word ook opgedeel in 'n konstante draaimoment gebied en 'n konstante drywingsgebied. (d) The safe  $P_m-\omega_m$  operating area of a separately excited DC motor can also be divided into a constant torque region and the constant power region.
- i. Teken die  $P_m-\omega_m$  bedryfsarea van die konstante draaimomentgebied en toon aan en verduidelik die waardes (simbolies) wat die bedryfsgrens definieer. [2] i. Sketch the  $P_m-\omega_m$  operating area of the constant torque region and indicate and explain the values (symbolically) that define the operating boundary. [2]
- ii. Teken die  $P_m-\omega_m$  bedryfsarea van die konstante drywingsgebied op (i) hierbo, en toon aan en verduidelik die waardes (simbolies) wat die bedryfsgrens in hierdie gebied definieer. [2] ii. Sketch the  $P_m-\omega_m$  operating area of the constant power region on (i) above, and indicate and explain the values (symbolically) that define the operating boundary. [2]

# Formules

$$\begin{aligned}\sin(\alpha+\beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \sin(\alpha-\beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \cos(\alpha+\beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \cos(\alpha-\beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta\end{aligned}$$

$$\begin{aligned}(fg)' &= f'g + fg' \\ \left(\frac{1}{g}\right)' &= -\frac{g'}{g^2} \\ \left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2}\end{aligned}$$

$$f(\theta_a) = a_v + \sum_{n=1}^{\infty} (a_n \cos(np\theta_a) + b_n \sin(np\theta_a)) d\theta_a$$

$$a_n = \frac{4p}{\pi} \int_0^{\frac{\pi}{2p}} f(\theta_a) \cos(np\theta_a) d\theta_a$$

$$b_n = \frac{4p}{\pi} \int_0^{\frac{\pi}{2p}} f(\theta_a) \sin(np\theta_a) d\theta_a$$

$$\oint_S \mathbf{B} \cdot d\mathbf{a} = 0$$

$$\int_S \mathbf{B} \cdot d\mathbf{a} = \phi$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = Ni = \mathcal{F}$$

$$\mathbf{B} = \mu\mathbf{H}$$

$$\mathcal{F} = \phi\mathcal{R}$$

$$\mathcal{R} = \frac{\ell}{\mu A}$$

$$\mu = \mu_0 \mu_r$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$B_m = \mu_R (H_m - H'_c) = B_r + \mu_R H$$

$$\mu_R = \frac{B_r}{H'_c}$$

$$(Ni)_{equiv} = -H'_c d$$

$$\mathcal{R}_{equiv} = \frac{d}{\mu_R A_m}$$

$$L = \frac{N^2}{\mathcal{R}}$$

# Formulas

$$\lambda = N\phi$$

$$L = \frac{\lambda}{i}$$

$$e = \frac{d\lambda}{dt}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$W_{fld}(\lambda, \theta_m) = \frac{1}{2} \frac{\lambda^2}{L(\theta_m)}$$

$$T_{fld} = \left. \frac{\partial W_{fld}(\lambda, \theta_m)}{\partial \theta_m} \right|_{\lambda}$$

$$W'_{fld}(i, \theta_m) = \frac{1}{2} L(\theta_m) i^2$$

$$T_{fld} = \left. \frac{\partial W'_{fld}(i, \theta_m)}{\partial \theta_m} \right|_i$$

$$\mathcal{F}_{ag}(\theta_{ae}) = \frac{4}{\pi} \left( \frac{N_{pha}(t)}{2p} \right) \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(n\frac{\pi}{2})}{n} k_{\omega_n} \cos(n\theta_{ae})$$

$$\theta_{me} = p\theta_m$$

$$\omega_e = p\omega_m$$

$$\omega_m = \frac{2\pi}{60} n_m$$

$$p = \frac{\text{poles}}{2}$$

$$n_m = \frac{60f_e}{p}$$

$$E_a = K_a \phi_d \omega_m$$

$$T_m = K_a \phi_d I_a$$

$$T_b = b\omega_m$$

$$T_J = J \frac{d\omega_m}{dt}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$