

Addisionele gegewens/Additional information

$$\Phi(t) = e^{\mathbf{A}t} = \text{Laplace}^{-1} \left\{ [s\mathbf{I} - \mathbf{A}]^{-1} \right\} \quad \Delta_{OL}(s) = |s\mathbf{I} - \mathbf{A}| = s^n + \alpha_1 s^{n-1} + \dots + \alpha_{n-1} s + \alpha_n$$

$$\mathbf{x}(t) = \Phi(t)\mathbf{x}(0) + \int_0^t \Phi(t-\tau)\mathbf{b} u(\tau) d\tau \quad \mathbf{X}(s) = \Phi(s)\mathbf{x}(0) + \Phi(s)\mathbf{b} U(s)$$

$$e^{-at} \Leftrightarrow \frac{1}{s+a}$$

$$t e^{-at} \Leftrightarrow \frac{1}{(s+a)^2}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{\text{kof}[\mathbf{A}]^T}{|\mathbf{A}|} = \frac{\text{adj}[\mathbf{A}]}{|\mathbf{A}|}$$

$$\text{kof}[\mathbf{A}] = \begin{bmatrix} a_{22}a_{33} - a_{23}a_{32} & a_{23}a_{31} - a_{21}a_{33} & a_{21}a_{32} - a_{22}a_{31} \\ a_{13}a_{32} - a_{12}a_{33} & a_{11}a_{33} - a_{13}a_{31} & a_{11}a_{32} - a_{12}a_{31} \\ a_{12}a_{23} - a_{13}a_{22} & a_{13}a_{21} - a_{11}a_{23} & a_{11}a_{22} - a_{12}a_{21} \end{bmatrix}$$

$$|\mathbf{A}| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$G(s) = \mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} + d$$

Tydgebied ontwerp/Time domain design:

$$t_r \approx \frac{1.8}{\omega_n}, \quad t_p = \frac{\pi}{\omega_d}, \quad 1\%t_s = \frac{4.7}{\zeta\omega_n}, \quad 2\%t_s = \frac{4}{\zeta\omega_n}, \quad s = -\sigma \pm j\omega_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$e^{j\omega T} = \cos \omega T + j \sin \omega T, \quad z = e^{sT}, \quad G_{ho}G(z) = \frac{z-1}{z} Z \left\{ \frac{G(s)}{s} \right\}$$

$$K_p = \lim_{z \rightarrow 1} D(z)G_{ho}G(z), \quad K_v = \frac{1}{T} \lim_{z \rightarrow 1} (z-1)D(z)G_{ho}G(z), \quad K_a = \frac{1}{T^2} \lim_{z \rightarrow 1} (z-1)^2 D(z)G_{ho}G(z)$$

$$e_{ss} = \frac{1}{1+K_p}, \quad e_{ss} = \frac{1}{K_v}, \quad e_{ss} = \frac{1}{K_a}$$

Z-Transform:

$$F(z) = \sum_{k=-\infty}^{\infty} f(k)z^{-k}, \quad f(k+1) = zF(z) - z f(0), \quad f(k-1) = z^{-1}F(z), \quad \lim_{k \rightarrow \infty} f(k) = \lim_{z \rightarrow 1} (z-1)F(z)$$

Bilineêre Transformasie/Bilinear Transformation:

$$s = \frac{2}{T} \left(\frac{z-1}{z+1} \right)$$

$F(s)$	$f(kT)$	$F(z)$
$\frac{1}{s}$	$1(kT)$	$\frac{z}{z-1}$
$\frac{1}{s+a}$	e^{-akT}	$\frac{z}{z-e^{-aT}}$
$\frac{a}{s(s+a)}$	$1 - e^{-akT}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$

Truwaartsverskil metode/Backward difference:

$$s = \frac{1}{T} \left(\frac{z-1}{z} \right), \quad \frac{1}{s} = T \frac{z}{z-1}$$

Tabelle met Z-transforms/Tables of Z-transforms

$F(s)$	$f(t); t \geq 0$	$f(kT)/f(k)$ $k = 0, 1, 2, \dots$	$F(z)$
—	—	Unit step $\mu(k)$	$\frac{z}{z-1}$
—	—	a^k	$\frac{z}{z-a}$
—	—	k	$\frac{z}{(z-1)^2}$
—	—	k^2	$\frac{z(z+1)}{(z-1)^3}$
—	—	$k a^k$	$\frac{az}{(z-a)^2}$
—	—	$a^k \sin \Omega k$	$\frac{(a \sin \Omega)z}{z^2 - (2a \cos \Omega)z + a^2}$
—	—	$a^k \cos \Omega k$	$\frac{z^2 - (a \cos \Omega)z}{z^2 - (2a \cos \Omega)z + a^2}$

Laplace transform

$$F(s)e^{-\Delta Ts}; 0 \leq \Delta < 1$$

z-transform

$$\mathcal{Z}[F(s)e^{-\Delta Ts}]; m = 1 - \Delta$$

$\frac{e^{-\Delta Ts}}{s}$	$\frac{1}{z-1}$
$\frac{e^{-\Delta Ts}}{s^2}$	$\frac{mT}{z-1} + \frac{T}{(z-1)^2}$
$\frac{2e^{-\Delta Ts}}{s^3}$	$T^2 \left[\frac{m^2 z^2 + (2m - 2m^2 + 1)z + (m-1)^2}{(z-1)^3} \right]$
$\frac{e^{-\Delta Ts}}{s+a}$	$\frac{e^{-amT}}{z - e^{-aT}}$
$\frac{e^{-\Delta Ts}}{(s+a)(s+b)}$	$\frac{1}{(b-a)} \left[\frac{e^{-amT}}{z - e^{-aT}} - \frac{e^{-bmT}}{z - e^{-bT}} \right]$
$\frac{ae^{-\Delta Ts}}{s(s+a)}$	$\frac{(1 - e^{-amT})z + (e^{-amT} - e^{-aT})}{(z-1)(z - e^{-aT})}$
$\frac{ae^{-\Delta Ts}}{s^2(s+a)}$	$\frac{T}{(z-1)^2} + \frac{amT-1}{a(z-1)} + \frac{e^{-amT}}{a(z - e^{-aT})}$